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The Nature and Frequency of Mathematical Discussion During Lesson Study That Implemented the CMI Framework

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THE NATURE AND FREQUENCY OF MATHEMATICAL DISCUSSION DURING LESSON STUDY THAT IMPLEMENTED THE CMI FRAMEWORK

by

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ABSTRACT

THE NATURE AND FREQUENCY OF MATHEMATICAL DISCUSSION DURING A LESSON STUDY THAT IMPLEMENTED THE CMI FRAMEWORK

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During a year-long professional development, the faculty members at an elementary school received instruction on mathematics and how to use the Comprehensive Mathematics Instruction framework. The instruction and the framework were consistent with the standards suggested by the National Council of Teacher of Mathematics (2000). This thesis analyzes the mathematical language used by three fifth-grade teachers who participated in lesson study to create a research lesson based upon the Comprehensive Mathematics Instruction framework.

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Chapter 1 – Introduction

Quality professional development in mathematics is necessary if teachers are to create the types of classroom environments envisioned by the National Council of Teachers of Mathematics (NCTM, 2000). The NCTM calls for "knowledgeable teachers [who are] continually growing as professionals" to teach "important mathematical concepts and procedures with understanding" (p. 3). Despite the NCTM's call for growth as professionals, professional development that meets the needs of teachers of mathematics is often not available (Loucks-Horsley, Love, Stiles, Mundry, & Hewsen, 2003). Lesson study is a form of professional development which is just beginning to become available to teachers of mathematics. It may have the potential to meet the NCTM's clarion call for knowledgeable and professionally growing teachers.

In recent years, lesson study has become an increasingly more popular venue for professional development of teachers of mathematics (Chokshi, 2004). As explained in further detail in Chapter 2, lesson study is a form of professional development originating in Japan. Teachers participating in lesson study collaboratively create a lesson, called a research lesson. Once the research lesson is created, it is then presented to students in a classroom where the participating teachers observe the effects of the lesson on students. The participating teachers subsequently reflect on, revise, and re-teach the research lesson. After the Third International Math and Science Study (TIMSS) and the publication of *The Teaching Gap* (Stigler & Hiebert, 1999), an increasing number of educators and administrators across the United States have taken an interest in lesson study (Lewis, 2002a).

Lewis (2002a) notes that some of the benefits of lesson study to teachers in Japan are that teachers think critically about goals in a content area, learn how other educators approach lessons, deepen their own subject matter knowledge, develop expertise in a subject and the capacity to learn from each other, and learn how students learn. Though the practice of lesson study in Japan is not required by national law, many teachers still consider it an essential component of their improved practice (Lewis, 2002a). Considering the benefits of lesson study in Japanese classrooms and the need for effective professional development in the United States, many educators hope that lesson study would be helpful to mathematics educators in the United States.

Since lesson study is relatively new to the U.S., there is still a need for meaningful research of the effectiveness of this type of professional development. There is a growing body of literature about lesson study, however, and this thesis is an effort to add to that growing body of knowledge. In this thesis, the conversations of elementary school educators participating in lesson study are recorded, transcribed, and analyzed.

By completing this thesis, I expect to understand the focus of the participants' conversations. Both the nature and the frequency of the topics discussed are of interest and therefore a mixed methods analysis is employed. That is to say, qualitative and quantitative methods of analysis are used on the collected data.

The research lesson which is the subject of this thesis is part of a larger research project conducted through a large private university and five cooperating school districts. In 2003, the Math Initiative Committee (MIC), a committee of university professors of education, mathematics, mathematics education, and instructional leadership; as well as administrators and mathematics specialists from five partner school districts, collaborated to "explore best practices for teaching and learning mathematics, and design a program for dissemination throughout the partnership for improving student mathematical understanding" (Walter, Peterson, Ridlon, & Hilton, 2006, p. 3). After numerous meetings, the MIC ultimately sculpted a framework for mathematics instruction.

The framework, known as the framework for Comprehensive Mathematics Instruction (CMI), was intended to aide teachers in teaching mathematics in a method consistent with the NCTM (2000) standards. Additionally, it "had to be flexible enough that it could be used with any curriculum in any of the partnership districts" (Walter et al., 2006, p. 9). The framework is described in greater detail in Chapter 2.

In order to test the framework, the MIC partnered with an elementary school close to the university and sponsored a professional development that lasted an entire school year. They sought a professional development program "furnishing… participants opportunities to construct for themselves more powerful, alternative understandings of learning, teaching, and disciplinary substance" (Schifter & Fosnot, 1993, p. 23). For that reason, the entire school's teaching staff received instruction in mathematics and instruction on teaching mathematics under the CMI framework. The biweekly professional development is explained in further detail in Chapter 3.

In the middle of the school year the teachers were afforded the opportunity to create mathematics lessons using the CMI framework. Grade-band groups of three or four teachers created research lessons to learn how to incorporate the CMI framework into their teaching practices. The fifth-grade teachers are the focus of this thesis.

This thesis analyzes the conversation of the fifth-grade teachers during the creation of their study lesson. Teacher conversations were expected to include items related to subject matter knowledge (SMK), pedagogical content knowledge (PCK), pedagogy, and management. In order to code the transcribed conversations, specific definitions for mathematics related SMK and PCK needed to be researched and established. Formal definitions of SMK, PCK, pedagogy, and management are presented in Chapter 2.

The CMI framework was intended to promote teaching consistent with the NCTM (2000) standards which promote teaching "concepts and procedures with understanding" (p. 3). Since lesson study originated in Japan and Japanese mathematics lessons are more conceptual than those generally found in the U.S. (Stigler & Hiebert, 1999), it is of interest to see whether the participants of this study focused their efforts on building a conceptual or a procedural lesson. For this reason, Skemp's (1978) definitions for relational and instrumental understanding are introduced for coding purposes.

Chapter 2 – Literature Review

The purpose of this chapter is to briefly describe the relevant components of teacher knowledge, such as SMK and PCK, which may be manifest during lesson study. The CMI framework and the structure of lesson study are also discussed in this chapter.

CMI Framework

The MIC sought to improve mathematics education by creating an instructional framework consistent with the standards proposed by the National Council of Teachers of Mathematics (NCTM, 2000). The framework was called the Comprehensive Mathematics Instruction framework, or CMI framework. The framework was created over a year's time of regular monthly meetings by the MIC.

In a pilot study during the second year of the MIC, the CMI framework was introduced to teachers at an elementary school. The introduction was done via a one-year biweekly professional development project. During the biweekly professional development sessions, the committee sought to improve mathematics instruction by first improving the mathematical knowledge of the participating teachers, and second, by training the participants to use the CMI framework. The format of the professional development is explained in detail in Chapter 3.

The CMI framework contains six components: (1) Launch, (2) Explore, (3) Discuss, (4) Extend, (5) Practice, and (6) Demonstrate Understanding (see Appendices A and B). Each component represents a different aspect of classroom teaching.

During the *launch* phase the teacher presents a task to the class. “Each task has a clear conceptual purpose tied to the standard core objective but is designed to allow multiple solutions or multiple paths to correct solutions” (Walter et al., 2006, p. 10).

In the *explore* phase the students work, either individually or in a group, on the task introduced during the launch. As the name indicates, and as situations arise, the students explore relevant mathematical ideas underlying the task.

The *launch* and *explore* phases of the CMI framework are consistent with the standards proposed by the NCTM. The NCTM (2000) states that “in effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. Well-chosen tasks can pique students’ curiosity and draw them into mathematics” (p. 18).

After the children explore the task, a teacher conducts a classroom discussion. This is formally known as the *discussion* phase of the framework. During a discussion, a teacher highlights different student solutions to the tasks by inviting students to present their solutions. The students then discuss the benefits of particular solutions. Mathematical discussion is also an important standard that the NCTM recommends for classrooms as evidenced by the following: “Are students’ discussion and collaboration encouraged? Are students expected to justify their thinking? ... Creating an environment that fosters these kinds of activities is essential” (NCTM, 2000, p. 18).

As students explore a particular topic they may have questions that would contribute to their further understanding. The *extend* component of the CMI framework allows for the extension of a task and provides opportunities for students to build further conceptual understanding. The purpose of the *practice* component is to promote procedural fluency which comes from continued practice of problems that are structurally and conceptually similar.

During each phase of the framework the teachers are able to evaluate understanding by listening to student conversations, observing student work, or by more formal methods. The *demonstrate understanding* component addresses the teacher's need to make informal and formal evaluations of student understanding throughout the entire process.

The faculty of an elementary school close to the university acted as a pilot group for implementation of the framework. The teachers became familiar with the CMI framework though instruction and modeling by university mathematics education professors. Members of the entire MIC occasionally taught, intervened, or presented topics related to the framework. In addition, the elementary school teachers gained experience using the CMI framework by creating research lessons. The process of creating a research lesson is formally known as lesson study.

Lesson Study

Loucks-Horsley et al. (2003) state that in the United States "professional development is still marginalized and mired in outmoded practices that serve neither teachers nor their students" (p. xvii). Another study found similar results:

> Once teachers reach the classroom, they often do not receive the support they need to keep their pedagogical skills and content knowledge current. Unlike in other professions, in education, few specific requirements and even fewer opportunities exist for teachers to engage in meaningful

professional development. (National Research Council Committee on Science and Mathematics Teacher Preparation [NRCC], 2001, p. 2)

Research conducted by SRI International (Shields, Esch, Humphrey, Young, Gaston, & Hunt 1999) found that in California schools a significant portion of time used for professional development is spent on instruction for new curriculum, instruction for the use of technology, assessment orientation, and classroom management. Slightly more than 50 percent of the teachers surveyed indicated that their professional development meetings addressed teaching methods specific to their subject area part of the time. Though the SRI International study focused on teaching practices in California, the NRCC (2001) had similar findings for the United States in general:

> Professional development for continuing teachers too often consists of a patchwork of courses, curricula, and programs and may do little to enhance teachers' content knowledge or the techniques and skills they need to teach science and mathematics effectively. (p. 33)

It is clear that there are professional development issues for U.S. teachers of mathematics that need addressing. Lesson study may be useful for U.S. mathematics teachers as it differs deeply from common U.S. professional development models (Stigler & Hiebert, 1999).

Lesson study is "a direct translation of the Japanese term *jugo kenkyu*, which is composed of two words: *jugo*, which means lesson, and *kenkyu*, which means study or research" (Fernandez & Yoshida, 2004, p. 7). It is a "well-defined process that involves discussing lessons that they have first planned and observed together. These lessons are called *kenkyu jugo*, which is simply a reversal of the term *jugo kenkyu* and thus literally means study or research lessons…" (Fernandez & Yoshida, 2004, p. 7).

The well-defined process of lesson study has the following components: (1) a collaborative planning of a research lesson, (2) observing the research lesson, (3) discussing the lesson, (4) revising the lesson, (5) teaching the revised version of the lesson, and (6) reflecting upon the research lesson (Fernandez & Yoshida, 2004).

During the collaborative planning stage of a research lesson, participants begin by choosing a goal and an academic subject or content area they will teach. Lewis (2002b) noted that in Japan "when considering the academic subject and topic for lesson study, teachers often: (1) target a weakness in student learning or development, (2) choose a topic teachers find difficult to teach, (3) choose a subject that has changed recently, for example, new content, technology, or teaching approaches that have been advocated, (4) concentrate on the study of Japanese and mathematics in alternate years, since these subjects account for much instructional time and can be fundamental to progress in other areas" (p. 28).

One particular version of lesson study is *konaikenshu* which translates to in-school training (Fernandez & Yoshida, 2004). In *konaikenshu* small groups of teachers from one school participate in the previously outlined lesson study process. Other venues for lesson study involve teachers from multiple schools. The lesson study which is the subject of this thesis could be classified as *konaikenshu*.

Lesson study is traceable to the early 1900's in Japan. By the mid-1960's lesson study as it is combined with *kenkyu jugo* was common in all of Japan (Fernandez &

Yoshida, 2004). Even though lesson study is not mandated by law, it is still a common practice in most elementary and middle grade schools in Japan (Lewis, 2002b).

Lesson study gained attention in the U.S. in 1999 with the publication of Stigler and Hiebert's (1999) *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. The book was written as a result of the video study portion of the 1995 TIMSS study. Before this study, lesson study was largely unknown in the United States (Lewis, 2002a).

The first research on lesson study in the United States is traceable to Yoshida and Stigler who began researching differences in Japanese and American teaching of mathematics in 1989 (Chokshi, 2004). The first published research on U.S. lesson study was based on work under the direction of Lewis, and to date there are at least 2,300 teachers in 32 states involved in lesson study (Chokshi, 2004). The MIC sponsored professional development was another venue for teachers to participate in lesson study. It was anticipated that the benefits of lesson study participation would allow the participants to improve their instructional practices in mathematics through the use of the CMI framework.

The benefits of using lesson study as a professional development are noteworthy. Lewis (2002b) stated that in Japan: "…the research lesson(s) provided an opportunity for these teachers to establish what knowledge was important, discover gaps in their own knowledge, and acquire the needed information" (p. 9). Through the process of creating research lessons, Japanese teachers can create an atmosphere of sharing and learning knowledge relevant to their subject area.

While preparing research lessons Japanese teachers discuss the following items: (1) a beginning problem (including exact numbers), (2) materials needed for the lesson, (3) anticipated student thoughts and solutions, (4) questions to promote student thinking, (5) space apportionment on the chalkboard, (6) how to address differing levels of mathematical preparation among the students, and (7) ending the lesson with opportunities for advancing mathematical thought (Yoshida as cited by Stigler & Hiebert, 1999).

Research Question

With Japanese lesson study providing its teachers with the venue they need for subject specific advancement and U.S. professional development in many cases not addressing the need altogether, it is of interest to find out if mathematics lesson study in the U.S. allows and encourages teachers to learn mathematics and principles of mathematics instruction. Therefore the research question addressed by this thesis is as follows: What is the nature and the frequency of mathematical discussion by in-service elementary school teachers participating in lesson study during the CMI professional development sessions? Answering this question will yield insights into whether the participants were engaged in the learning of mathematics and/or mathematics instruction.

The CMI professional development presented an atmosphere for the participants to create a research lesson by participating in lesson study. As they did so, they prepared, taught, revised, re-taught, and reflected on a mathematics lesson. It was expected that conversations would focus on both knowledge about mathematics and knowledge about mathematics as it pertained to teaching. For the purposes of this paper, conversation about mathematics (pertaining to teaching or not) is classified as mathematical discussion. The teachers also spoke about pedagogical issues independent of mathematics. Mathematics knowledge dependent and independent of pedagogy will be defined later.

Types of Mathematical Knowledge

Ma (1999) observed that Chinese teachers participating in lesson study deepen their subject matter knowledge. Yoshida (As cited by Stigler & Hiebert, 1999) noted that Japanese teachers engaged in lesson study spoke about a beginning problem, anticipated solutions, necessary materials, suggested teacher questions, space allotment on the chalkboard, and extensions. It is likely that U.S. teachers participating in lesson study would be similarly engaged in conversations pertaining to subject matter knowledge, student thinking, and classroom management issues.

The types of knowledge that the teachers cited by Ma (1999) and Yoshida (As cited by Stigler & Hiebert, 1999) used are also types of knowledge that U.S. teachers acquire and use in their practices. These are formally known as subject matter knowledge, pedagogical content knowledge, knowledge of classroom management, and knowledge of pedagogy. The remainder of this chapter is dedicated to the explanation of those types of knowledge as well as Skemp's (1978) relational and instrumental understanding.

Subject Matter Knowledge. Subject matter knowledge (SMK), often referred to as content knowledge, is knowledge about a subject. In this thesis SMK relates to the subject of mathematics. Leinhardt and Smith (1985) state that "subject matter knowledge includes concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curriculum presentation" (p. 247). Leinhardt and Smith's categorization of SMK includes both SMK as it pertains to mathematics and as it pertains to teaching mathematics. In reference to mathematics they state that SMK includes concepts, algorithmic operations, connections, and subsets of the number system. For purposes of this thesis, subsets of the number system are understood to mean subsets of the real numbers such as rational numbers, integers, positive integers, or irrational numbers.

Shulman (1986) states more broadly that SMK is the amount and organization of one's knowledge. Shulman also asserts that a person with SMK knows the key topics of one's subject and why those topics are important. The amount of one's knowledge would take into consideration the items that Leinhardt and Smith (1985) considered essential to SMK, such as algorithmic procedures and subsets of the number line. The organization of knowledge is not addressed in Leinhardt and Smith's definition of SMK.

In a later paper, Wilson, Shulman, and Richert (1987) state that SMK includes both substantive and syntactic knowledge:

> Subject matter knowledge includes both the substantive and syntactic structures of the discipline. The substantive structures include the ideas, facts, and concepts of the field, as well as the relationships among those ideas, facts, and concepts. The syntactic structures involve knowledge of the ways in which the discipline creates and evaluates new knowledge. (p. 118)

Wilson, Shulman, and Richert add the category of syntactic knowledge to Leinhardt and Smith's (1985) previously existing definition of SMK. Their definitions of substantive and syntactic knowledge are analogous to Schwab's (1964).

Schwab (1964) described substantive knowledge as knowledge about the structure of a discipline and the philosophies and paradigms that guide inquiry in that subject. An example of substantive knowledge in mathematics is knowledge of what mathematics is. Mathematics is considered by some to be about problem solving, whereas by others mathematics is considered to be modeling.

Syntactic knowledge, according to Schwab (1964), is the knowledge of the evidence that guides inquiry in a particular field. One who possesses syntactic knowledge of mathematics would recognize patterns and phenomena which would in turn guide inquiry.

Grossman, Wilson, and Shulman (1989) believe that there are three dimensions to SMK: (1) content knowledge, (2) substantive knowledge, and (3) syntactic knowledge. Content knowledge for them is knowledge of "factual information, organizing principles, [and] central concepts… An individual with content knowledge can identify relationships among concepts in a field as well as relationships to concepts external to the discipline" (Grossman, Wilson, & Shulman, 1989, p. 27). The factual information aspect of Grossman et al.'s content knowledge is analogous to previous definitions of SMK because factual information may include algorithms, guiding principles, and other central topics to a subject. Grossman, Wilson, and Shulman also include organizing principles in their definition of content knowledge. Whereas Grossman, Wilson, and Shlman consider content knowledge a subset of SMK, other authors refer to SMK and content knowledge interchangeably (Feimann-Nemser, 1990; Van Dooren, Verschaffel, & Onghena, 2002).

SMK is also described in terms of number of credit hours that a person took in a particular subject or the score one achieves on a standardized test (Begle, 1972). However, "knowledge of subject matter encompasses more than what is typically measured in standardized multiple choice tests, and certainly more than is reflected in the number of classes that someone has taken" (Grossman, Wilson, & Shulman, 1989, p 25). Whereas standardized tests may manifest portions of one's SMK, Grossman, Wilson, and Shulman assert that an evaluation of one's SMK is not appropriately based upon standardized tests or credit hours. Van Dooren, Verschaffel, and Onghena (2002) state that SMK may even manifest itself in one's problem solving behavior.

This thesis defines SMK using the descriptions above. SMK includes concepts, algorithmic operations and connections between different algorithmic procedures. SMK also includes knowledge of student understanding and errors; however, as stated in the next section, this type of SMK is referred to hereafter as pedagogical content knowledge. SMK is the amount and organization of one's knowledge. It includes knowledge about the key topics of one's subject and why those topics are important. SMK also includes knowledge about how topics should be organized, which is also considered part of pedagogical content knowledge. SMK includes substantive and syntactic structures of a discipline. Contained in SMK is the ability to recognize relationships within mathematics and relationships to items external to mathematics. In addition to problem solving behavior, this thesis looks for SMK to manifest itself in the conversations of in-service elementary teachers who are participating in lesson study.

Though SMK for teachers is knowledge of both mathematics and pedagogy, for the purposes of this thesis, SMK is used exclusively to describe knowledge of mathematics which is not used in conjunction with pedagogy. Other categories are introduced in this chapter to describe SMK as it pertains to pedagogy (see Figure 1 at the end of Chapter 2).

Pedagogical Content Knowledge. As mentioned previously, SMK for teachers of mathematics is not only knowledge of mathematics, but also knowledge of how mathematics should be taught. In 1986, Shulman introduced the term pedagogical content knowledge (PCK) to describe the pedagogy aspect of SMK. There are differing viewpoints about PCK as to whether or not it is a subset of SMK. Grossman (1995) states that PCK is contained in SMK, and Even (1993) states that "teachers' pedagogical content knowledge is influenced by their subject-matter knowledge. However, the interrelations between the two are still very much unknown" (p. 97).

If we consider SMK as subject matter knowledge for teachers, then it follows that PCK may be considered a subset of SMK because SMK for teaching encompasses a knowledge of a subject as well as knowledge of pedagogy. For the purposes of this thesis, PCK is considered a subset of SMK.

Shulman (1986) based his description of PCK on the writings of Dewey (1962), Scheffler (1965), Green (1971), Fenstermacher (1978), Smith (1980), and Schwab (1983). What follows is a succession of those authors' ideas contributing to Shulman's definition of PCK.

Knowledge of a subject matter and knowledge of pedagogy are sometimes treated as two different subjects. University mathematics education programs, for example, place students in mathematics classes to learn mathematics, and teaching courses to learn pedagogy with students from other disciplines. John Dewey (1962) critiqued such compartmentalization when he wrote that "scholastic knowledge is sometimes regarded as if it were quite irrelevant to method. When this attitude is even unconsciously assumed, method becomes an external attachment to knowledge of subject matter" (pp. 327-328). Dewey recognized that there is a tendency to dissociate methods of pedagogy from the subject matter one is teaching. Subject matter, however, is inextricably associated with the pedagogy necessary for the learning of that subject matter.

Scheffler (1965) also recognized the correlation between SMK and pedagogy when he wrote that "education is concerned to transmit not only what we know, but the manner of knowing, that is, out of approved standards of competence in performance, in inquiry, and in intellectual criticism" (p. 2). According to Scheffler there are approved standards and competencies associated with subject matter. So it is not only important what one knows, but how one comes to know it.

Green (1971) expounded upon this point when he said the following:

> Teaching is almost always aimed at getting someone to learn. Indeed, it is hard to imagine any other motive for teaching. If any explanation is without logical fault, it is a good explanation in one respect. But it may also be a bad explanation to give at a certain time to people who may not be equipped to understand it. For example, an explanation in physics may be sound in every logical respect, appropriate for a graduate seminar, and yet be a bad explanation to give children in the fourth grade. In teaching, typically, we are concerned not only that our reason, evidence,

> conclusions, and explanations be good in a logical sense, but also that they be good in a heuristic sense…. (p. 8)

According to Green, it is possible to have sound logic and reasoning in one's teaching and yet not be effective in teaching from a pedagogical point of view. A teacher expounding a subject in the manner portrayed by Green would be unaware of the mathematical needs of his students.

Fenstermacher (1978) notes that teaching even the most basic skills requires that the teacher know and address the needs of the students:

> …The way we now handle basic skills in schools may impair the learner's ability to pursue advanced knowledge. Basic skills may be basic in the sense that they are needed to function at a most elementary level in society, or they may be basic in the sense that these skills are needed to access more advanced knowledge… If we understand basic skills in the latter, more clearly educative, sense, it should be obvious that teaching them is not a simple, routine, low-order task. However, even if the basic skills were simple routines, the conclusion that teaching simple skills is a simple task is unsupported. (p. 174)

According to Fenstermacher, teaching basic skills is not a basic activity. It requires consideration for what and how a student may learn in the future. Similarly, Schwab (1983) suggests that teaching should be guided by what is known, as well as what is yet to be known. In conjunction with Green's (1971) assertion that teachings meet the needs of the students being taught, it may be inferred that a student's needs are closely tied to the subject matter at hand.

A student's need in relation to subject specific teaching was noted by Smith (1980) in a report to the United States government when he wrote the following:

> The interaction between teacher and student… is guided by considerations stemming partly from the teacher's knowledge of how to proceed and partly from the fact that there is subject matter involved in her interaction. The teacher interacts with the student through a body of knowledge, and the student in turn interacts with the teacher through the same material. (p. 81)

As Smith noted, the interactions of students and teachers are guided by the fact that there is a specific subject matter at hand. This indicates that student and teacher interactions may differ from subject to subject.

Teacher-student interactions are as well shaped by a teacher's knowledge of student knowledge. Fenstermacher (1971) noted the importance of this when he wrote that "an important part of the sophistication required for teaching resides in the ability to recognize the learner's source of error and confusion, not the source of success" (p. 174).

Based upon the previous authors' works, Shulman (1986) introduced the phrase pedagogical content knowledge. PCK is the specific body of knowledge unique to teachers that encompasses the content and the pedagogical needs of the students. Shulman states that SMK is the amount and organization of knowledge in the teacher whereas PCK is subject matter knowledge for teaching and is of particular interest because "…it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interest and abilities of learners, and presented for instruction" (p. 8).

To this point of the section on PCK, a motivation and a description of Shulman's (1986) definition of PCK were presented. Since Shulman's introduction of PCK, it has been adapted specifically to mathematics and used in a variety of ways. The remainder of this section is devoted to comparing different definitions of mathematics specific PCK and similarly defined teacher specific knowledge. From the compared definitions, a concise definition of PCK will be presented for use in this thesis.

Ball and Bass (2003) have written extensively on "mathematical knowledge for teaching" and they state that "what constitutes necessary knowledge for teaching remains elusive [and that] the nature of knowledge required for teaching remains underspecified" (pp. 4-5). They consider PCK and mathematical knowledge for teaching as differing one from another. Ball and Bass (2000) describe having PCK as being prepared for irregularities that arise in a typical mathematical task or situation, but "knowing mathematics for teaching must take into account both irregularities and the uncertainties of practice, and must equip teachers to know the contexts of the real problems they have to solve" (p. 90).

A context, as defined in the American Heritage Dictionary (Pickett et al., 2000) is a setting or a circumstance where an event occurs. Thus, according to Ball and Bass (2000), mathematical knowledge for teaching should include knowledge of circumstances and events where real problems are solved. When teachers understand the real life circumstances or events where a problem is solved they may then better adapt their teachings to the diverse interests of their students. Knowing "how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners" (Shulman, 1986, p. 8) is part of pedagogical content knowledge. Thus, for the purposes of this thesis, Ball's mathematical knowledge for teaching is considered to be a subset of Shulman's PCK.

Another form of teacher knowledge introduced in recent years is Ma's (1999) "knowledge package." In *Knowing and Teaching Elementary Mathematics*, Liping Ma (1999) describes a knowledge package as "a network of procedural and conceptual topics supporting or supported by the learning of the topic in question" (p. 124). In the case of mixed number subtraction (the topic of the participants' research lesson in this thesis), for example, a knowledge package would include the conceptual principal that mixed numbers may be written and grouped in different ways ($2\frac{3}{4} = \frac{11}{4} = 1\frac{7}{4}$), and a knowledge of different algorithms for solving mixed number subtraction (i.e. converting both the minuend and subtrahend to mixed fractions before finding a difference, or regrouping the fraction in the minuend to allow subtraction of the fraction portion of the subtrahend). A network of procedural and conceptual topics supporting a topic in question determines how "problems or issues are organized" (Shulman, 1986, p. 8), which is a necessary part of PCK. That is to say that the same concepts that constitute a knowledge package are also contained in Shulman's definition of PCK and therefore Ma's knowledge package is not disjoint from PCK.

For this thesis, PCK is considered a blending of content and pedagogy and is also contained in SMK. The blending of content and pedagogy that is considered PCK in this thesis takes into account student thinking. This includes how an educator might organize and present a topic, and common student errors. Knowledge of pedagogy is SMK for teaching and therefore PCK is SMK applied to teaching (see Figure 1 at the end of Chapter 2).

Relational Understanding and Instrumental Understanding. Merely categorizing the mathematical discussions of the participants as SMK and PCK may not fully describe the nature of the mathematical discussions or the nature of the mathematics in the research lesson. This section introduces the need for sub-categories related to procedural and conceptual knowledge it then presents the motivation for the use of Skemp's (1978) relational and instrumental understanding as sufficient sub-categories.

Stigler and Hiebert (1999) state that mathematics lessons in the United States are largely procedural, that mathematical concepts are stated by the teachers rather than developed, and that there is a lot of switching between topics. In contrast, Stigler and Hiebert note that mathematics teaching in Japan focuses on developing concepts and there is more coherence between mathematical topics.

The fact that U.S. lessons are not as conceptual in nature as Japanese lessons is noteworthy because research indicates that lessons which aid students in gaining conceptual understanding have long-lasting benefits.

> Understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting… When students practice procedures they do not understand, there is a danger they will practice incorrect procedures, thereby making it more difficult to learn correct ones. (Mathematics Learning Study Committee, 2001, p. 122)

A research lesson that aids students in understanding would be very valuable to the student participants of the lesson. Therefore it seems worthwhile to determine if and when the participants of this research focused on conceptual and procedural topics when creating the research lesson. The categorizations of SMK and PCK do not illustrate whether the participants of the CMI lesson study focused on procedures or concepts when creating their research lesson. Thus, a distinction between conversations that are procedural in nature or conceptual in nature needs to be made.

In order to distinguish between conceptual and procedural conversations it is important to distinguish what is meant by conceptual knowledge and procedural knowledge. Hiebert and Lefevre (1986) characterize conceptual knowledge as "knowledge that is rich in relationships. It can be thought of as a connected web of knowledge… in which the linking relationships are as prominent as the discrete pieces of information" (pp. 4-5). Davis (1986) describes conceptual knowledge as the type of knowledge that he has of his own home:

> Probably my knowledge of the layout of the house I live in is… mainly conceptual. I can start anywhere in reconstructing a mental representation of the rooms, doors, stairways, windows and other features, and I can answer any reasonable question. Of course, I have lived in the house for more than 10 years.
>
> By contrast, my knowledge of how to walk between two buildings on an unfamiliar university campus is, at first, sequential. I do not at first see how everything relates to everything else. I could not draw an accurate map, and some of my routes may be far from optimal… (pp. 232-233)

Hiebert and Lefevre (1986) categorize procedural knowledge in two parts. "One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks" (p. 6). Procedural knowledge may also be recognized as being sequential in nature (Davis, 1986).

The coding of the TIMSS project used the categories of procedural and conceptual mathematics to distinguish between the types of mathematics lessons taught in classrooms in different countries (Jacobs et al., 2003). Similar information is sought in this study. The nature of mathematical discussion by the participants possibly was procedural in nature, conceptual in nature, or both.

As the procedural and conceptual knowledge that the CMI participants used in the creation of their research lesson is of interest, this thesis includes categories which are intended to measure emphases that the teachers place on these two types of knowledge. As Silver (1986) states, "distinctions between conceptual and procedural knowledge do not constitute sharp, impenetrable barriers" (p. 181). It is expected that there may not be any sharp distinctions between conceptual and procedural knowledge in this study either. For this reason, it may be very difficult to code for conceptual and procedural knowledge.

Skemp's (1976) relational and instrumental understanding are the two categories that this paper uses instead of conceptual and procedural understanding. According to Skemp, relational understanding is "knowing both what to do and why" (p. 20), and instrumental understanding is a "rule without reason" (p. 20). It is expected that using relational and instrumental understanding instead of conceptual and procedural understanding will aid in avoiding ambiguities when coding.

One such ambiguity might be trying to assess whether a participant is trying to teach a concept, procedure, or both, when sharing an algorithm with another participant. Merely coding for procedural versus conceptual understanding would lend such a presentation to various interpretations. It may be considered procedural because an algorithm is generally sequential in nature. However, it may be considered conceptual if explanation of the algorithm is offered.

For the purposes of this thesis, relational understanding is knowing both what one is doing and why one is doing it. Instrumental understanding is knowing a rule without knowing why or how it works.

Returning to the algorithm example, a participant sharing an algorithm with another participant would be considered instrumental conversation if the algorithm were shared without discussing the underlying concepts of the algorithm. If the algorithm were shared in a manner intending to aid the learner in understanding the underlying concepts, then the participant would have been participating in a relational conversation.

Relational and instrumental understandings are applied to both SMK and PCK as subcategories. For example, one may work within the realm of SMK while using either relational or instrumental understanding. Similarly, one may also work with PCK and as well use either relational or instrumental understanding (see Figure 1 at the end of Chapter 2).

In this thesis relational and instrumental sub-categories of both PCK and SMK refer explicitly to the nature of the participants' conversations and not to their thinking. If a participant presents an answer to a problem by performing an algorithm, then his sentences would be classified as instrumental. If a participant presents an answer to a problem by performing an algorithm and describing why that algorithm works, then his sentences would be classified as relational.

Classroom Management and Pedagogy

Classroom teaching of mathematics involves SMK and PCK as they relate to mathematics. It also requires organizational aspects that are unique to the environment in which the learning takes place. Since the research lesson of the participants is intended for a fifth-grade class, discussion during the creation of the research lesson necessarily contained conversation about issues unique to this environment.

SMK for teachers includes knowledge of pedagogy. The American Heritage Dictionary (Pickett et al., 2000) defines pedagogy as “the art or profession of teaching.” For the purposes of this paper pedagogy refers to general statements about teaching which are not specific to mathematics. Pedagogy which is specific to mathematics was previously defined as PCK. An example of a pedagogical issue would be a discussion about a student’s academic background. An exchange of such information could be important to the teaching of that child, but would not necessarily deal with mathematics.

Participant conversations concentrated at times on maintaining an appropriate and orderly learning environment. For this purpose, a category of classroom management is introduced.

> When teachers talk about being in charge or being in control, they are referring to the management function. When they speak of classroom control, they are referring to their ability to maintain order and sustain pupil attention. Since order is a stabilizing influence in the classroom,

maximizing security and minimizing distractions, management is often equated with maintaining classroom control. (Froyen, 1993, p. 32)

As Froyen states, classroom control is often equated with classroom management.

Classroom management is an aspect of “the art or profession of teaching” (Pickett et al., 2000) and is therefore a subcategory of pedagogy. For the purposes of this thesis, classroom management is designated as any statement whose purpose is to implement classroom control, maintain safety, or diminish distractions. Classroom management that applies to how a mathematics lesson should be taught is a subset of PCK (see Figure 1 in the next section).

Summary

Figure 1 is a diagram outlining the relationships between the types of knowledge described in this chapter. There are two types of SMK: (1) SMK for mathematics, and (2) SMK for pedagogy. The SMK for pedagogy which is exclusive of SMK for mathematics is considered “pedagogy” for this thesis. The intersection of SMK for pedagogy and SMK for mathematics is PCK. PCK is the shaded region in Figure 1. Relational understanding and instrumental understanding are subsets of both SMK and PCK. Management is a subset of both pedagogy and PCK.

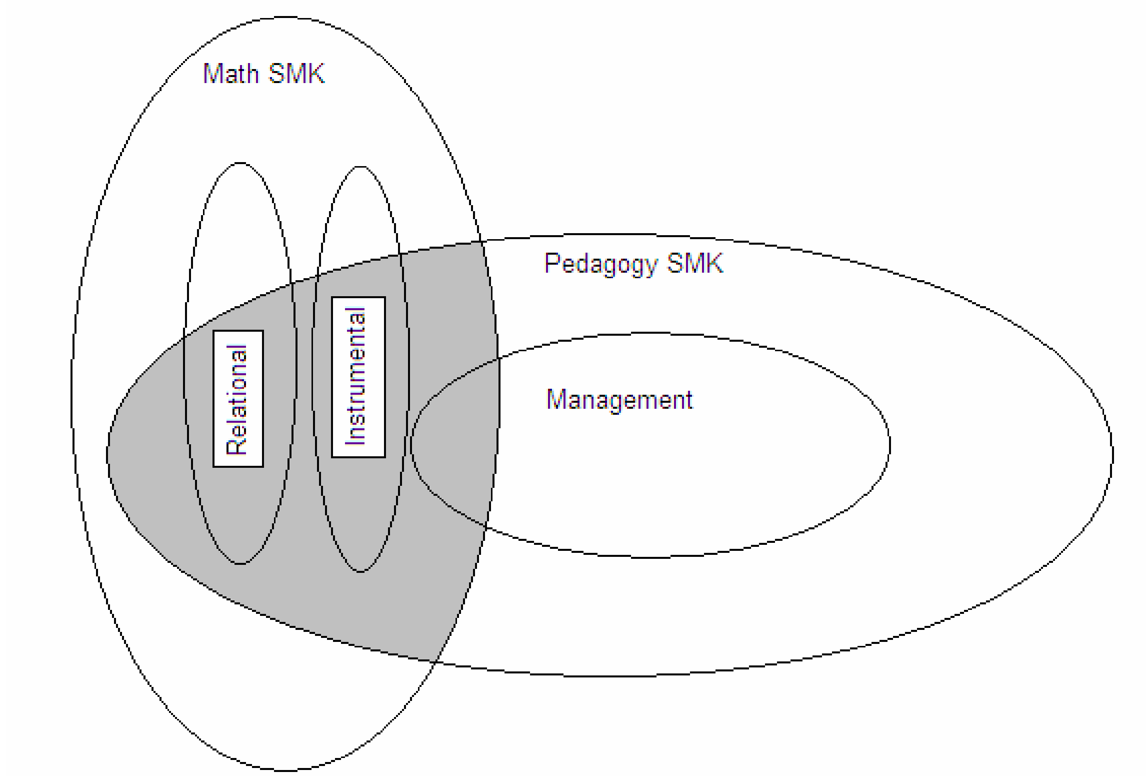


Figure 1. Knowledge diagram where PCK is shaded.

This chapter described the interrelationships of various types of teacher knowledge. The nature of mathematical knowledge, which is a part of this thesis, is SMK for mathematics and its subsets. The method for identifying the frequency of mathematical discussion is unveiled in Chapter 3.

Chapter 3 – Methodology

Methods

The data analysis in this thesis is done using mixed methods. According to Creswell (2003), a mixed methods approach is one which uses components of both qualitative and quantitative research. The qualitative analysis, as described in the coding section of this chapter, involves categorizing the content of the participants' conversations about mathematics. The quantitative analysis, as detailed in the analysis section of this chapter, entails performing a statistical analysis of the data collected during the coding process. Reading through the coded transcripts and looking for patterns in the participants' conversations was also an important aspect of the qualitative research. A qualitative analysis was necessary to determine what types of conversations the participants had while planning a research lesson. A quantitative analysis was necessary in order to compare the frequency of occurrence for the categories of conversation.

Background

During the 2003/2004 school year, members of the MIC created the CMI framework (see Appendices A and B). (Both the MIC and the CMI framework are described in detail in Chapters 1 and 2.) An elementary school near the sponsoring university was then chosen to pilot the new framework. During the 2004/2005 school year, the entire faculty at the pilot school received instruction on the implementation of the framework. The instruction for the framework occurred during bi-monthly professional development meetings.

In addition to receiving instruction on implementation of the framework, the faculty received mathematics instruction from professors of mathematics education belonging to the sponsoring university. The task-based instruction reflected the type of instruction proposed by the CMI framework and also offered the teachers opportunities to study various elements of mathematics in the state's core curricula. The professional development also included a lesson study component intended to help the faculty implement the CMI framework in their own classrooms.

The professional developments between the months of August and November were devoted to the instruction of mathematics and the CMI framework. During the December break the participants read the book *Lesson Study: A Handbook of Teacher-Led Instructional Change* (Lewis, 2002b) as a preparation for the lesson study component of the professional development which occurred from January to March. During the months of January to March teachers continued to receive mathematics instruction as well. Table 1 details the timeline of the professional development as it pertained to the focus group. The lesson study schedule differed slightly for the other participants insomuch that some of their research lessons were presented on different dates.

Table 1

Professional development and lesson study overview

Date	*Action*
August 16 & 17	Two day orientation to the professional development which included mathematics instruction.
September 13	Mathematics and framework instruction

September 20	Mathematics and framework instruction
October 4	Mathematics and framework instruction
October 18	Mathematics and framework instruction
November 1	Mathematics and framework instruction
November 15	Mathematics and framework instruction
November 29	Mathematics and framework instruction
January 10	Review of assigned lesson study reading Participants began researching and formulating research lesson goals
January 24	First lesson study goal chosen and stated Mathematics instruction
February 7	Mathematics and framework instruction Research lesson preparation
February 23	First research lesson presented in Lew's classroom
February 28	Research lesson revised Mathematics and framework instruction
March 1	Revised research lesson presented in Camilla's classroom Review of research lesson with observers
March 14	Brief report of research lesson made to all participants by Camilla Mathematics and framework instruction
March 28	Mathematics and framework instruction
April 11	Mathematics and framework instruction
April 25	Mathematics and framework instruction

Setting and Participants

Before implementation of the lesson study the teachers worked in heterogeneous grade-level groups of four to six people. Homogeneous grade-level groups replaced the heterogeneous groups for creation of the research lessons. The focus group of this study, the fifth-grade teachers, contained three teachers: Lew, Janice, and Camilla. During the revision of the research lessons, Yolanda, an administrator for the school, replaced Lew who was absent.

Lew graduated from a private university with a degree in elementary education. His teaching experience spans 30 years during which time he taught at the second, third, and fifth-grade levels. In addition to the two semesters of mathematics for elementary school teachers required for his bachelor's degree, Lew attended a district sponsored mathematics intensive professional development in 2003.

Janice also graduated from a private university with a degree in elementary education and had 26 years of elementary school teaching experience at the time of this study. Twenty-two of the 26 years were spent teaching fifth-grade. Her mathematics training also included the two semesters of mathematics for elementary school teachers for her bachelor's degree and the district's mathematics intensive professional development in 2003.

Camilla was a graduate of a state university with a bachelor's degree in elementary education. At the time of this study she was pursuing a master's degree in education. She was a second year teacher of the fifth-grade. Camilla's mathematics training included the following courses: Algebraic Foundations of Arithmetic, Mathematics Pedagogy, Utah State Core Curriculum, and Technology for Mathematics Instruction.

Dr. S was one of the mathematics education professors involved in the CMI professional development. He aided the teachers in their research lesson creation by acting as an advisor throughout the lesson study process.

On January 10th in preparation for the research lesson, the teachers discussed their reading of *Lesson Study: A Handbook of Teacher-Led Instructional Change* (Lewis, 2002b). To begin creation of the research lessons, the participants chose lesson study goals and lesson study topics related to fractions. To facilitate understanding of the goal component of the research lesson, Dr. S related the overarching goals discussed in Chapter 2 of Lewis' book to the NCTM's (2000) process standards. Lew, Camilla, and Janice chose a preliminary research lesson goal on January 10th and finalized it on January 24th. More detailed discussion of the research lesson goal is found in Chapter 4.

Preparation for the research lessons continued on January 24th and February 7th. On February 23rd, Lew presented the first version of the research lesson to his fifth-grade class. The observers of that research lesson were Janice, Camilla, and Dr. S.

On February 28th, Janice, Lew, and Yolanda revised the research lesson. As mentioned previously, Lew was absent during that professional development session. On March 1st, Camilla taught the revised research lesson to her fifth-grade class. In addition to Lew, Janice, and Dr. S, the first-grade teachers and a visiting professor from Japan observed Camilla's lesson. Following Camilla's lesson, she and the observers met to reflect upon the lesson.

My Role as a Researcher

I filmed each of the professional development sessions. In addition to filming the presentation of the research lessons, I also filmed Lew's classroom three other times during the 2004/2005 school year for the MIC research. I limited my interaction with the participants to filming and an occasional casual conversation within the school. At no time did I attempt to instruct the teachers or influence their learning or teaching.

Data Collection and Transcription

I filmed each phase of the lesson study using digital video equipment. In addition to filming, the teachers submitted some copies of their personal notes and their students' work associated with the research lesson.

Video equipment was used because of the facility it offered for recording and analysis of verbal communication, written communication, and gestures. I transcribed the data using a personal computer DVD player and Microsoft Excel. The Excel spreadsheet contained columns for the time, a speaker's name, quotes, the coding, and memos. Some of the participants' written sentences and notes were placed in brackets. Each sentence by a participant was placed in a cell in the "quote" column of the spread sheet (see Table 2 at the end of this chapter).

Coding

As some portions of each professional development were allotted to creation of the research lesson and other portions were allotted to mathematics instruction, I viewed all of the pertinent video tapes to see which portions of the tapes contained segments of research lesson preparation. Later I transcribed each of the identified sections of tape by placing sentences from the participants in the Excel spread sheet described above. One sentence (or incomplete sentence) was placed in each cell of the spreadsheet. After transcription of the research lesson, a second party checked the entire transcript for accuracy.

The basic unit of analysis for the data was a sentence. There are differences between written sentences and verbal sentences. Those differences altered the way the data was coded. There were frequent interruptions of one participant by another as well as frequently unfinished sentences and therefore following the flow of the conversation was very important to the coding process.

In order to transcribe accurately and maintain the integrity of the flow of conversation, interruptions were inserted where they occurred. Subsequently a full sentence often times spanned a number of rows on an Excel spreadsheet. Whenever an interruption occurred, causing a sentence to span multiple rows, each portion of the sentence was coded individually according to the whole meaning of the sentence.

Similarly, whenever a participant was interrupted and did not complete their sentence, the sentence was coded according to the flow of conversations. If, for example, all of the other sentences surrounding the interruption were coded as SMK, then the unfinished sentence was assumed to be SMK also unless the nature of the sentence clearly indicated otherwise. By using this method of coding, any errors in calculating quantitative results were consistent throughout the data.

I coded using the categories outlined in Chapter 2: (1) SMK, (2) PCK, (3) relational understanding (R), (4) instrumental understanding (I), (5) classroom management (M), and (6) other pedagogical issues (P) (see Appendix C and Figures 2 and 3 below).

Since PCK (the shaded portion of Figure 2) is a subset of SMK, for ease of coding, PCK is used exclusively instead of in conjunction with SMK. Also, management issues that are a subset of PCK were double coded as PCK, M to distinguish them from management, which is a subset of Pedagogy but is not mathematics related. Figure 3 shows the coding abbreviation for each region of the diagram.

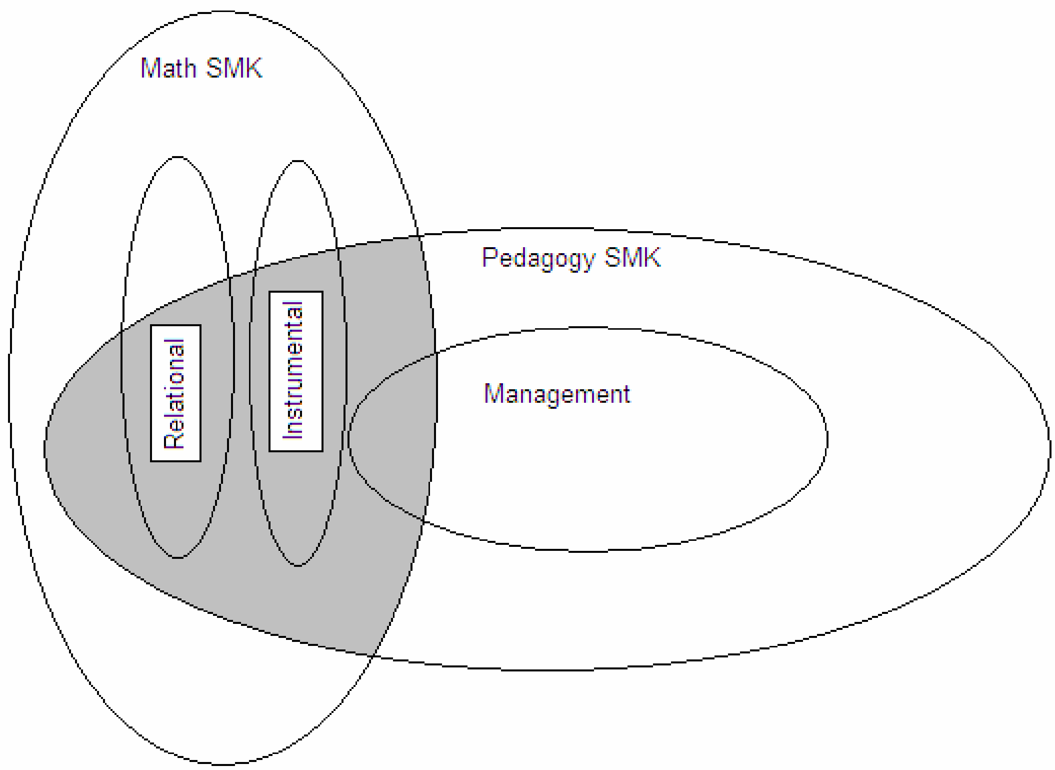


Figure 2. Knowledge diagram where PCK is shaded.

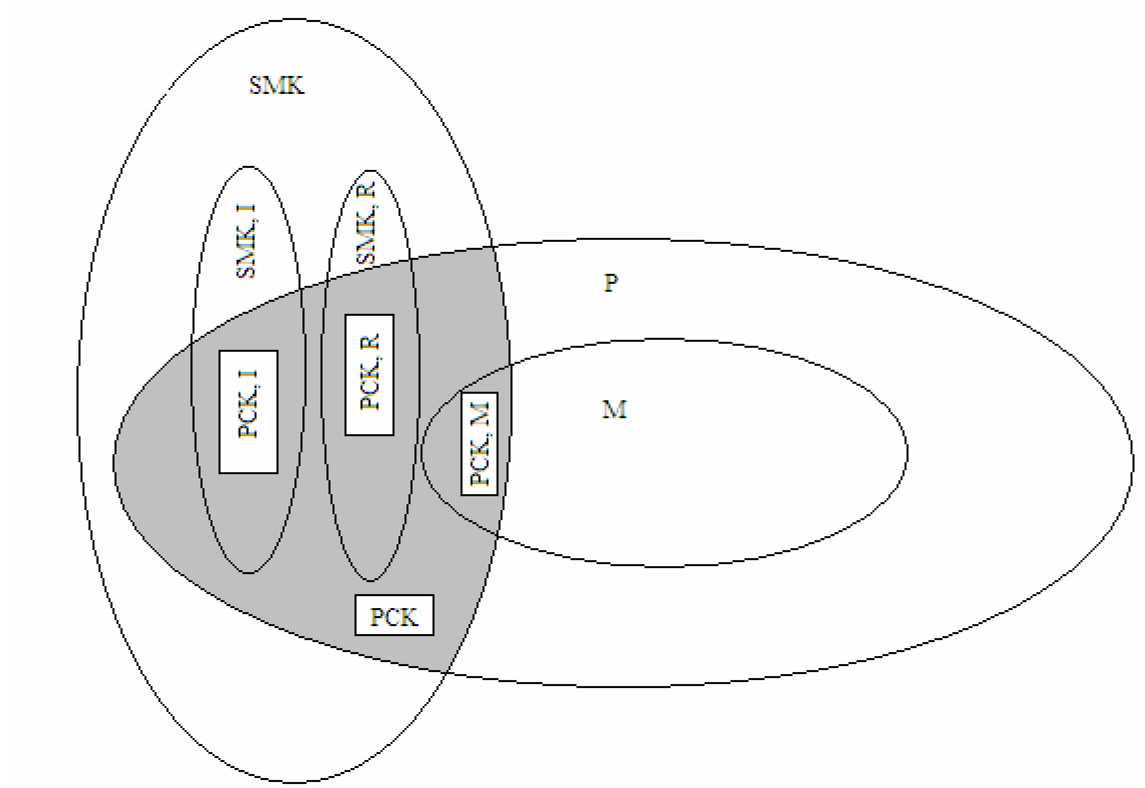


Figure 3. Coding diagram.

Participant discussion of student solutions often focused on student errors. PCK conversations about student errors were double coded as instrumental (PCK, I) if only the mistaken steps in a solution were the focus of conversations. PCK conversations about student errors were double coded as relational (PCK, R) if conversations about student mistakes focused on underlying misconceptions which caused a mistake.

Open Coding

While coding for the six categories suggested previously, three new categories became necessary in order to describe some of the unexpected conversations which occurred during the creation of the research lesson: (1) personal, (2) protocol, and (3) discussion (see Appendix D).

At various stages during the creation of the research lesson, the participants engaged in personal conversation. For the purpose of this thesis, personal conversation is conversation "concerning a particular person and his or her private business, interests, or activities" (Pickett et al., 2000). Every instance of personal conversation was coded as "Personal."

Occasionally the participants engaged in conversation about issues related to protocol. Items of protocol were, for instance, questions about what format they needed to use as they wrote their lesson plan, or what the professional development instructors expected them to do during the creation of their research lesson. These items, though important to the creation of the research lesson, did not directly deal with the teaching of mathematics nor teaching in general.

Because of concerns expressed by Dr. S to the MIC regarding the teachers' struggles to orchestrate a discussion, I coded all instances where the discussion phase of the lesson was the topic of conversation. Any time the participants engaged in conversation about the discussion portion of the lesson, a code of "D" was applied. The reason for the new code is described in further detail in Chapter 5.

The addition of the three new codes constitutes part of what is known as open coding. Open coding is the "the analytic process through which concepts are identified and their properties and dimensions are discovered in data" (Strauss & Corbin, 1998, p. 101). After the initial coding process I inspected the data for examples of the codes. By extracting examples of the codes, I noticed patterns in the conversations pertaining to the codes. Those patterns are listed and described in Chapter 5. After recognizing patterns, I then searched and analyzed the data to either confirm or disprove the hypothesized patterns.

Table 2

Sample Coding

Time	*Speaker*	*Quote*	*Code*	*Memo*
3.15	Janice	Ok, so what's gonna be our carefully written lesson study?	PCK	
	Lew	Our what?	PCK	
	Lew	What's our what?	PCK	
	Camilla	Carefully written lesson study goal.	PCK	
	Janice	Lesson study goal.	PCK	
	Lew	Our carefully written lesson study goal is...	PCK	
	Janice	To investigate…	PCK	
	Lew	… by the end of the lesson you'll be able to rename…	PCK	
	Janice	Ok, but remember how we had first of all, talked about last time, engaging all students…	PCK	
	Camilla	Engaging all students.	PCK	
	Janice	…and improving math self-esteem by…?	PCK	
	Janice	Remember that was part of that lesson study when	PCK	

	we read, that you look at…	
Camilla	Ya.	PCK
Janice	…what goal it is you're trying to work on.	PCK
Janice	In math, one of the things we're trying to do is improve math self-esteem.	PCK

Analysis

Quantitative analysis of the data was done using Excel. The total number of times each code appeared was compared to the total number of codable sentences. A code's frequency of use was determined by listing its percentage of overall occurrence.

Chapter 4 – Data and Analysis

The purpose of this chapter is to present data and results. Sample conversations are presented to evidence the appropriateness of the codes described in Chapter 3. Codes (as shown in Appendices C and D) will follow each direct quote given in the text. In order to put some of the data in perspective, an overview of the lesson is presented before the samples are given.

Research Lesson Overview

The planning of the research lesson began on January 10th at the first professional development session after the December break. On that day Dr. S began the session by inviting the participants to discuss any new ideas they learned while reading a book by Lewis (2002b). The participants responded by immediately focusing on goals as evidenced by Lew: "I think we should formulate goals for student's learning in the long… term development" (P). Camilla and Janice responded favorably and Janice suggested that they "work together to consider the long term goals for students" (P).

As stated on page nine, the research lesson goal is an important part of the research lesson. A unique feature of a research lesson goal is that it focuses on long tem qualities that students should have (Lewis, 2002b). This idea is new to many U.S. educators (Lewis, 2002b) and it was new to Camilla, Janice, and Lew as well.

Later the participants received specific instruction on goals and goal setting. Dr. S explained goal setting by relating long term and short term goals to the NCTM's (2000) process and content standards. The process standards "address the processes of problem solving" (NCTM, 2000, p. 7) and are (1) problem solving, (2) reasoning and proof, (3) communication, (4) connections, and (5) representation. The content standards "describe mathematical content goals" (NCTM, 2000, p. 7) in (1) number and operations, (2) algebra, (3) geometry, (4) measurement, and (5) data analysis and probability.

The participants were instructed to have two types of goals: overarching goals, like those found in the NCTM's (2000) process standards, and goals tied to the content area of fractions. At this point Lew began to discuss a concern of his about teaching mathematics:

> I don't… I'm not… I sitting here and I'm thinking, ok I can teach kids how to reduce fractions. I could teach kids how to find common denominators and all that stuff. My biggest problem in the classroom over thirty full years is how to get the kids that just sit there because they think it's too hard… and so they're not even going to try to engage in a thought. It's those things that cause my scores to go down. (PCK)

As the participants continued to discuss Lew's concern that certain children do not engage in mathematical thought, Dr. S invited the participants to address that concern through their research lesson goal. Lew, however, was not satisfied that the process standards met his children's needs when he responded that "there's a lot of things in the process that aren't even on there. For instance: math self-esteem" (PCK). Subsequently the participants decided upon "improving math self-esteem" as a principle part of their research lesson goal. They felt that children who were not engaging in mathematics lessons needed improved mathematics self-esteem. The content that they first chose to address in their research lesson was renaming fractions.

On January 24th when the participants convened for another professional development session they looked at their lesson study goal in relation to the timeline of the remainder of their school year. After reviewing their calendar they realized that they were to teach the research lesson nearly six weeks in the future. The participants then proceeded to look for content that would be more suitable for their time schedule. After perusing several lessons and lesson ideas in their teaching manuals they decided to teach a lesson on mixed fractions because of the difficulty students incur in learning the subject:

Janice: Subtraction is a lot harder (PCK).

Lew: Cause you have to regroup (PCK, R).

Janice: You have to regroup and it's way hard for 'em to do (PCK, R).

Their research lesson goal then became "to improve math self-esteem while learning how to subtract mixed fractions." It is noteworthy that by choosing a lesson which is traditionally hard for the children to understand, the participants "target(ed) a weakness in student learning and development" (Lewis, 2002b, p. 8) which is a common practice among Japanese educators involved in lesson study.

With the subtraction of mixed fractions as a content goal for the research lesson, the participants needed to choose a problem that would serve as a launch for the lesson. Initially they chose $7\frac{1}{8}-1\frac{3}{4}$ from a lesson manual because it required borrowing. They then tried to create a meaningful launch based upon those numbers:

Janice: Ok, what would happen, what would happen if we literally put a problem like this up on the board? (PCK) [Pointing to $7\frac{1}{8}-1\frac{3}{4}$]

Lew: Ya, let's just do that. (PCK)

Janice: You just put a problem like that up on the board. (PCK)

Lew: Just like she did. (PCK) (Referring to a video of a lesson which they had recently watched)

Janice: Ya, just exactly like that – and with the answer though. And they have to come out with how we did it. (PCK)

Camilla was interested in building student understanding and asked "What if we gave 'em the manipulatives and made 'em prove it?" (PCK, R). Janice suggested drawing pictures as a manipulative: "What we could do is have them draw it to how we got the answer" (PCK, R), and Camilla entertained the idea of using graham crackers as manipulatives after reading about a graham cracker activity in a lesson book. Though the participants continued to discuss an appropriate launch for $7\frac{1}{8}-1\frac{3}{4}$, they did not decide upon one that day.

In addition to discussing an appropriate launch for $7\frac{1}{8}-1\frac{3}{4}$, the participants also suggested possible student solutions to the same problem. Later in the day they were given even more opportunity to meditate upon anticipated student solutions to subtraction of mixed fractions problems. The mathematics and framework training on January 24th was a worksheet meant to aid the participants in anticipating student solutions for a subtraction of mixed fractions problem (see Appendix E). Though the time allotted to the participants for creating the research lesson had officially ended, the worksheet proved to be invaluable to their research lesson preparation. For that reason, their time with the worksheet was included as part of the data.

When Lew, Janice, and Camilla reviewed their lesson study goal on February 7th, they decided that they needed to alter the problem to better match the classroom manipulatives, such as pattern blocks. In order to facilitate student understanding with pattern blocks the participants exchanged the problem $7\frac{1}{8}-1\frac{3}{4}$ for $3\frac{1}{6}-1\frac{5}{6}$. Camilla explained the participants' motivation for the choice of $3\frac{1}{6}-1\frac{5}{6}$ explicitly when questioned after the presentation of the revised research lesson:

> We picked, we picked the first two, uh, with same denominators but forcing them to borrow… That was our main thing. And originally our, the worthwhile mathematical task that we made up was different and we felt like it needed to be more conducive to what they had for manipulatives (PCK).

With the goal intact (improving math self-esteem through subtraction of mixed fractions), and a problem to accompany a launch ($3\frac{1}{6}-1\frac{5}{6}$), Camilla, Janice, and Lew began to prepare the research lesson.

The launch they chose was as follows: "If you have $3\frac{1}{6}$ reams of paper and you use $1\frac{5}{6}$ reams of paper, how much paper do you have left?" The overarching research lesson goal still remained the same: "To engage all students and improve math self-esteem through teaching subtraction of mixed fractions."

The first presentation of the research lesson was in Lew's room on February 23rd. Camilla and Janice met again on February 28th to review and edit the research lesson. Since Lew was absent during the professional development session of February 28th, Yolanda, a school administrator, aided in the editing process. Two major changes were made to the lesson on that day.

Out of consideration for the manipulatives, a problem involving pizza was traded for the problem involving reams of paper. It was suggested that pattern blocks might better represent pizza than reams of paper. The launch associated with $3\frac{1}{6}-1\frac{5}{6}$ was altered to: "I came to school with $3\frac{1}{6}$ pepperoni pizzas. Mr. L ate $1\frac{5}{6}$ of them. How many pizzas are left for the class party?" The second major change to the lesson was to rewrite the launch so that the students would not consider $3\frac{1}{6}$ and $1\frac{5}{6}$ as 3 groups of $\frac{1}{6}$ and 1 group of $\frac{5}{6}$ respectively. That was a problem that arose in the first research lesson that they did not want to focus on in the revised research lesson.

The following day, March 1st, Camilla taught the revised research lesson in her classroom. Directly after the class, Camilla, Lew, Janice, Dr. S, the four observing first-grade teachers, and a visiting professor from Japan adjourned to a nearby classroom to review the research lesson.

During the professional development session on the 14th of March, Camilla presented to the rest of the faculty some of her group's reflections on the research lesson. The conversations of the six aforementioned days where the participants either created or reflected upon the research lesson constituted the data for this research (see Table 3 for an abbreviated description of the six days).

Table 3

Synopsis of Six Days of Coded Data

Dates	*Action*
January 10th	Conversation about lesson study Initial goal of renaming fractions
January 24th	Goal for subtraction of mixed fractions with $7\frac{1}{8}-1\frac{3}{4}$ Anticipated student responses
February 7th	Launch altered for manipulatives: $3\frac{1}{6}-1\frac{5}{6}$
February 23rd	First research lesson by focus group was taught in Lew's classroom
February 28th	Yolanda aids in lesson revision Wording of launch altered
March 1st	Focus group's revised research lesson taught in Camilla's room
March 1st	Reflection of lesson with all observers
March 14th	Camilla reports to the faculty

In Chapter 2 it was stated that Japanese teachers participating in lesson study discussed the following items: (1) a beginning problem (including exact numbers), (2) materials needed for the lesson, (3) anticipated student thoughts and solutions, (4) questions to promote student thinking, (5) space apportionment on the chalkboard, (6) planning for different student mathematical preparation for the lesson, and (7) ending the lesson with opportunities for advancing mathematical thought (Yoshida as cited by Stigler and Hiebert, 1999).

Janice, Lew, and Camilla's discussions included many of the same items. They spoke about a beginning problem (including exact numbers), materials that they would need for the lesson, anticipated student thoughts and solutions, a few questions to promote student thinking, and student preparation for the lesson. Instead of space apportionment on a chalkboard they chose to use an overhead projector in both classrooms. The participants did not end their lessons with opportunities for advancing mathematical thought.

The remainder of this chapter is devoted to the presentation of the data samples and the results of the coding. Each code is briefly described and a sample of the associated data presented. A percent of overall occurrence is included in each category and summarizing tables are presented at the end of the chapter (see Tables 4 through 6).

Subject Matter Knowledge

As stated in Chapter 2, SMK in this thesis is defined as knowledge of concepts, algorithmic operations and connections between different algorithmic procedures. In a general sense SMK and PCK are separate codings insofar as PCK is SMK applied to teaching. SMK manifests itself relatively few times.

In the following example, Camilla and Lew were discussing anticipated student responses for a problem involving subtraction of mixed fractions. The conversation occurred while they were collaborating on a worksheet which was distributed to all the professional development participants on January 24th. As mentioned previously, though the time officially set apart for the creation of the research lesson had ended, these conversations were valuable to Camilla, Janice, and Lew's research lesson, and are therefore included.

Camilla suggested that one method of solving $2\frac{1}{4}-1\frac{3}{4}$ would be to convert both of the mixed fractions to improper fractions ($\frac{9}{4}-\frac{7}{4}$) and then find the difference of the numerators ($\frac{2}{4}$). Lew responded “I never would have thought of changing them into improper fractions and then subtracting” (SMK). Realizing that they had not considered that as a method of solution for their lesson study question, Camilla, Lew, and Janice began to look again at their lesson study problem ($7\frac{1}{8}-1\frac{3}{4}$) and consider whether or not it could be solved in the same manner:

Camilla: But on the other one does it work? (SMK)

Camilla: Ya, it does. (SMK)

Camilla: It will work every time. (SMK)

Lew: Does it? (SMK)

Lew: On the one we were saying? (SMK)

Camilla: Ya. (SMK)

Camilla: It'll work every time. (SMK)

Lew: No, well, but first you got to get a common denominator. (SMK, R)

Janice: You gotta get a common denominator first. (SMK, R)

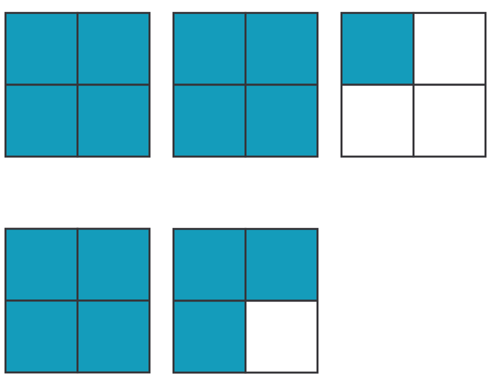
Lew: This has a common denominator already. (SMK, R)

The conversation was coded as SMK because, for a moment, Janice, Lew, and Camilla stopped focusing on student thinking and in turn questioned their own knowledge of mathematics. Camilla asked herself if the method of changing to improper fractions would work for $7\frac{1}{8}-1\frac{3}{4}$. She responded to her own question in the affirmative just before Lew asked the same question himself. There was a turning point in the conversation, however, when Camilla said “it’ll work every time” (SMK). Lew’s response that “first you got to get a common denominator” (SMK, R) was a transition into relational SMK because he addressed the underlying idea of why the method worked.

SMK conversation accounted for 2.2% of the total conversations by the participants (see Table 4).

Subject Matter Knowledge with Relational Understanding

Another example of relational SMK was also found during the time when the participants collaborated on the January 24th worksheet. While suggesting anticipated student responses to $2\frac{1}{4} - 1\frac{3}{4}$ Camilla drew three squares where each square was partitioned into four equal parts. Directly below those three squares she drew two more squares and partitioned each into four equal parts. Two of the upper squares were completely shaded, and one-fourth of the remaining square was shaded. One of the lower squares was completely shaded and three of the four partitions of the other lower square



were shaded (see Figure 4 below).

Figure 4. Camilla's first drawing.

Camilla proceeded to cross out one upper shaded partition of one square and one lower shaded partition of one square. She repeated that process until all of the lower partitions were crossed out (See Figure 5).

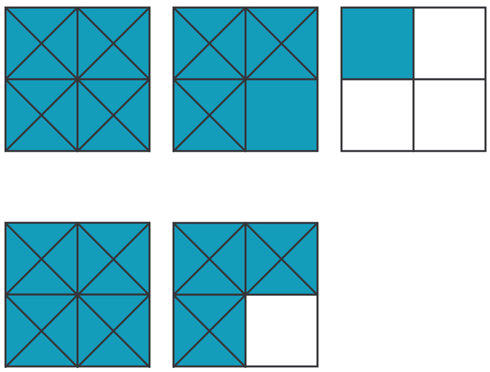


Figure 5. Camilla's second drawing.

Dr. S, who was at the table at the moment, called this a comparison model. He then proceeded to describe the difference between a comparison model and a "take away" model:

Dr. S: It's a little different than take away. (SMK, R)

Camilla: Right. (SMK, R)

Dr. S: Cause take away, you have this stuff and you just start pulling stuff away. (SMK, R)

Lew: Right. (SMK, R)

Dr. S: The difficult thing in that is knowing, keeping track of how much you're taking away. (SMK, R)

Dr. S: That's why the comparison model is you can line 'em up and say ok, it's just tellin' you how much to take away. (SMK, R)

Camilla: Right. (SMK, R)

Lew: I like that. (SMK, R)

Lew: Okay, good. Groovy. (SMK, R)

This example is coded as SMK because the participants are learning about mathematical concepts for their own knowledge. It is coded as relational because Skemp's (1976) categorization of what one is doing and why is applicable. Stated more specifically, they were instructed on why a comparison model is different from a take away model.

The last three sentences, the first by Camilla and the two by Lew are as well coded as SMK and relational because they indicate an affirmation by both Camilla and Lew that they understood Dr. S' instruction.

For coding purposes, statements like Lew's pronouncement of "right" or "Okay, good, Groovy" are considered sentences even though they are grammatically incomplete when transcribed and presented in written form.

Fewer than two percent (1.2%) of the total sentences were both SMK and relational. The relational SMK coding accounted for 53.5% of the total SMK sentences (see Table 5).

Subject Matter Knowledge with Instrumental Understanding

During the creation of the study lesson, there were no sentences that were coded as both SMK and instrumental. Such a sentence might have occurred during a conversation between participants who were seeking to enhance their own knowledge of mathematics by stating rules or algorithms about subtraction without the reasoning behind them.

Pedagogical Content Knowledge

Recall from Chapter 2 that PCK is a blending of content and pedagogy that takes into account student thinking. A conversation about PCK might be about how a topic should be organized or presented. It might also be about student errors or student understanding. A very general definition of PCK used for this thesis is that it is SMK applied to teaching.

The following example of PCK conversation occurred during the creation of the research lesson. The participants were trying to make additional problems to use for the *extend* or *practice* components of the CMI framework. As they considered possible problems to present to the class, they also considered how the students might feel about the problems.

> **Camilla**: Like if we're trying to increase their their, um, self-esteem about math, I guess not every math lesson could be, can have graham crackers and be super fun… (PCK)
>
> **Lew**: Right. (PCK)

Camilla: But I guess what makes me think of that is if we're trying to increase math self-esteem, or even just the way they feel about math and the enjoyment that they have with math… (PCK)

Lew: Well you can give them something that's seven and one-eighth… (PCK)

Camilla: Ya, so that's what I was trying to think 'cause… (PCK)

Lew: …of something and one and three-fourths of something. (PCK)

Janice: We can try the graham crackers too. (PCK)

Janice: We can, we have graham crackers. (PCK)

Camilla: Because I was just thinking like I don't know if my math self-esteem is going to be improved if my teacher throws somethin' up on the board and says do it. (PCK)

Camilla: Like, I I think I would feel intimidated and, you know what I'm sayin'? (PCK)

Part of the participants' goal was to improve mathematics self-esteem and Camilla wanted to make sure that the lesson about subtraction of mixed fractions would indeed improve mathematics self-esteem. Camilla continually brought the problems to a level she thought would be interesting to the children. For example, when she said "because I was just thinking like I don't know if my math self-esteem is going to be improved if my teacher throws somethin' up on the board and says do it", she was taking into account both content and pedagogy.

Lew's sentences were also coded as PCK because he was suggesting questions that could be used with graham crackers. When he said "well you can give them something that's seven and one-eighth," he was suggesting a way to make a problem that dealt with the graham cracker as a manipulative. This was also a blending of content and pedagogy.

Janice mentioned that "We can try the graham crackers too. We can. We have the graham crackers." These sentences were considered PCK because they contributed to the conversation by giving support to Camilla's suggestion that they use graham crackers in the lesson for pedagogical reasons.

PCK accounted for 73.6% of the total sentences spoken during the lesson study process (see Table 4).

Pedagogical Content Knowledge with Relational Understanding

At various times during the creation of the research lesson the participants engaged in conversation related to PCK which were also relational in nature. Upon deciding that the skill they desired to teach was subtraction of mixed fractions, the participants began discussing what types of activities might best be used to launch such an activity. The following is a portion of the conversation that occurred after it was suggested that a problem might be introduced by merely writing it on the board along with its answer:

Camilla: What if we gave 'em the manipulatives and made 'em prove it? (PCK, R)

Lew: Ya. (PCK, R)

Janice: Well or to draw it. (PCK, R)

Camilla: Make 'em do it. (PCK, R)

Janice: What we could do is have them draw it to how we got the answer.

(PCK, R)

In the above example, the participants were discussing a mixture of content and pedagogy. The sentences were therefore coded as PCK. The content they were discussing was subtraction of mixed fractions. The pedagogical aspect was how they would invite students to understand the subtraction problem.

Camilla suggested that they give the students manipulatives and make them prove, or justify, their answer. By Camilla requesting that the students justify their thinking with manipulatives, she was asking that they justify how something works and why. This type of justification is relational reasoning.

Lew's sentence "ya" was coded as PCK and relational as well because he expressed agreement with Camilla's suggestion to require the children to justify their thinking. Similarly, Janice's suggestion that the children draw a picture was another way of inviting children to justify their thinking. Camilla's response to "make 'em do it" was an expression of agreement with Janice's suggestion.

Another type of PCK conversation which was relational in nature also dealt with student misconceptions. As mentioned on pages 25 and 25, conversations about student mistakes were considered relational in nature when the participants discussed the underlying student misconceptions which motivated the error. One such conversation occurred on January 24th as the participants collaborated to think of student misconceptions for the worksheet problem $2\frac{1}{4}-1\frac{3}{4}$. Lew and Janice suggested that students might alter the fraction $2\frac{1}{4}$ to be $1\frac{11}{4}$, thereby changing the problem to $1\frac{11}{4}-1\frac{3}{4}$:

Lew: So they'll go like this and then they'll, then they'll make than one and bring an eleven over so then they'll get… (PCK, R)

Janice: Eight-fourths. (PCK, R)

Janice: I did that one. (PCK, R)

Janice: Because they do that all the time. (PCK, R)

Janice: And even though you'll say does that even make sense? (PCK, R)

Janice: They don't… (PCK, R)

Lew: Well it's taking what they learned from… (PCK, R)

Janice: Ya. (PCK, R)

Lew: ...regular… (PCK, R)

Dr. S: From whole numbers. (PCK, R)

Lew: Ya, from whole numbers. (PCK, R)

In this example, student misconceptions were the focus of conversation so PCK was an appropriate code. Though Lew and Janice's explanation of a student mistake seemed to begin very instrumentally when they explained how a student would convert $2\frac{1}{4}$ to $1\frac{11}{4}$, relational was the appropriate coding because the participants discussed the underlying misconception that would cause the error.

Lew recognized that the error would stem from a misapplication of base ten regrouping when he began to explain "well it's taking what they learned from… regular…" at which time Dr. S aided in applying formal vocabulary by stating "from whole numbers." It is clear from Lew's sentences that he was explaining an underlying conceptual misunderstanding so his sentences were coded as relational. Janice's agreement, "ya," was coded as relational because she was demonstrating agreement with

Lew. Dr. S' sentence was as well coded as relational because he aided in completing Lew's sentence.

Relational PCK accounted for 7.8% of the total sentences made by the participants. The percentage of PCK sentences which were relational in nature was 10.6 of the PCK (see Table 6).

Pedagogical Content Knowledge with Instrumental Understanding

While preparing the study lesson the participants were asked to think of as many anticipated student responses as they could for their chosen question. At the moment of the request the participants' lesson still focused on the problem $7\frac{1}{8}-1\frac{3}{4}$. A portion of their conversation follows:

Camilla: What if they just do it this way and they go four times two is eight. (PCK, I)

Camilla: Three times two is six. (PCK, I)

Camilla: So then they have… (PCK, I)

Janice: And five-eighths, six and five-eighths. (PCK, I)

Camilla suggested that one possible student response would involve rewriting $1\frac{3}{4}$ as $1\frac{6}{8}$. Their suggestion was that a student might solve $7\frac{1}{8}-1\frac{6}{8}$ by subtracting the 1 from the 7 and subtracting $\frac{1}{8}$ from $\frac{6}{8}$. A student response would then be $6\frac{5}{8}$.

The type of reasoning in the conversation above is coded as PCK because the participants considered student thinking and student errors. The student errors that the participants considered were instrumental errors. Instrumental thinking involves using rules without reasoning (Skemp, 1978). An instrumental error, therefore, could involve a misuse of an algorithm because of a lack of understanding for the rules underlying the algorithm. This would be the case with the type of subtraction described above. Therefore, the coding for the excerpted conversation was classified as PCK and instrumental.

Sentences coded as both PCK and instrumental comprised 1.5% of the total sentences, and 2.1% of the PCK sentences were instrumental (see Tables 4 and 6).

Personal

Various items distracted the participants from focusing on their lesson. Some common distractions were the presence of the camera, a malfunctioning writing utensil, or the food they were consuming during the professional development. When the participants became distracted and conversed about items about a "person and his or her private business, interests, or activities" (Pickett et al., 2000), their conversations were coded as "personal." An example of personal conversation occurred when, in the process of revising the research lesson, Yolanda aided Janice in realizing that the pen Janice used was ruining her paper:

Yolanda: It's ruining your other page. (Personal)

Janice: I know it. (Personal)

Camilla: I know. (Personal)

Yolanda: If you care. (Personal)

Janice: That's why I don't like these pens. (Personal)

The conversation was coded as personal because it centered on issues that were personal in nature and did not contribute to the research lesson. Personal conversations accounted for 9.1% of the total sentences made by the participants (see Table 4).

Protocol

Occasionally the participants conversed about items which related to protocol. For the purpose of this thesis, protocol was considered to be an expectation. How a lesson plan should be arranged so as to meet a professor's expectations, for example, would be considered protocol. One example of protocol occurred after the second teaching of the research lesson while the participants engaged in reflection.

One of the lesson observers, Annie, noticed that Camilla used the phrase "worthwhile mathematical task" frequently in her class. Annie wondered if she should have been using the same phrase in her classroom:

Annie: Are they… I noticed that you were saying worthwhile mathematical task. (Protocol)

Annie: Is that, um, language that you use all the time with 'em? (Protocol)

Camilla: Mmm huh. (Protocol)

Lew: (Inaudible) helping to improve math phobia.

Camilla: I use that everyday. (Protocol)

Annie: Is that something every math teacher should be saying? (protocol)

When Annie asked if the phrase "worthwhile mathematical task" was something "every teacher should be saying," she wanted to know if as a teacher she was expected to use the same phrase in her classroom. The coding of protocol was most applicable to this situation.

Lew's incomplete sentence "helping to improve math phobia" was not coded because it was not understood. After using various speakers and headphones it was not clear what Lew had said and therefore a correct coding of Lew's sentence was not possible. Protocol sentences comprised 2.8% of the total sentences (see Table 4).

Discussion Component of the CMI Frameowrk

As stated in Chapter 2, the CMI framework contained the following components: (1) Launch, (2) Explore, (3) Discuss, (4) Extend, (5) Practice, and (6) Demonstrate Understanding. While observing the creation and presentation of the study lesson, it was noted that the participants did not spend much time preparing to orchestrate a discussion in their classrooms with their students. (Each of the examples in this chapter stemmed from conversations about other components of the CMI framework). In order to analyze how much time they spent preparing or talking about the *discussion* component of the CMI framework, the coding of "discussion" (D) was used for every sentence which pertained to the *discussion* component of the framework.

The following is an example of a conversation about orchestrating a classroom discussion. Dr. S began the conversation by asking the participants in which order they would invite their students to present their solutions:

Lew: Why don't we just start with pictures? (PCK, D)

Camilla: Same (PCK, D)

Janice: Pictures, pictures, exactly (PCK, D)

Camilla: I definitely think pictures because once they make a connection with pictures… (PCK, R, D)

Janice: Then it's easier. (PCK, R, D)

Lew: Go to the abstract. (PCK, R, D)

Camilla: … then it's easier to do it the other way. (PCK, R, D)

Janice: That's right. (PCK, R, D)

Camilla: Which so they learn that the other way is more efficient, so they're able to do that better, but if it's an abstract thought, they don't even know what a fourth of two and one-fourth is. (PCK, R, D)

Lew: Ya, so I would say always start with the… (PCK, R, D)

Camilla: Ya, I think automatically, ya. (PCK, R, D)

Lew: Always start with concrete to abstract. (PCK, R, D)

It is clear that the sentences pertained to the orchestration of a classroom discussion. The coding of PCK was also applicable because the participants were engaged in conversation about the how the lesson should be organized and presented. A portion of the sentences are also coded as relational because the participants were discussing how best to organize the conversation so as to invite student understanding.

It would have been equally as worthwhile to code the participants' sentences about the launch and explore phases of the CMI framework. However, the planning of the launch and the explore phases were so intertwined that it was often difficult to code for either one explicitly.

The sentences about the discussion comprised 12% of the total sentences (see Table 4).

Management

Sentences about classroom control, maintaining safety, or diminishing distractions, were coded as management. Sentences which were coded as management occurred as Camilla, Janice, and Yolanda collaborated to revise the research lesson on February 28th. A number of observers would enter Camilla's classroom on March 1st to observe the research lesson and Camilla explained how she prepared her students to behave in a classroom full of observers:

Camilla: So they know tomorrow that people are coming to watch. (M)

Camilla: I just told 'em that they're the best class in the school and people wanna watch 'em do math. (M)

Yolanda: Of course. (M)

Camilla: And that's why. (M)

Yolanda: Uh huh. (M)

Janice: Ya right. (Personal)

Camilla: We are the best class in the school. (Personal)

Janice: Next to mine. (Personal)

Camilla: Anyway. (Personal)

Camilla: So, so they know that. (M)

Camilla: They know they have to be on their best behavior. (M)

During the revision of the research lesson Camilla mentioned various times that she was nervous about having observers in her classroom watching her teach. The preceding quotes demonstrate Camilla's efforts to maintain classroom control while under observation by other teachers. Therefore Camilla's sentences are coded as management and so are Janice and Yolanda's affirmations of her sentences. The brief bantering by Janice and Camilla is coded as personal.

Though the conversation among Janice, Camilla, and Yolanda was about the presenting of the research lesson, the conversation was not coded as PCK management because the management issue Camilla was addressing was how to promote student behavior during a rigorous classroom observation and not how to promote classrrom behavior in a way specific to mathematics teaching.

During the lesson study process there were conversations about management that were unique to the subject of mathematics and how a mathematics lesson should be taught. An example of management PCK conversation occurred as Camilla, Janice, and Yolanda discussed the effects of group size and the effect it had on student use of mathematics manipulatives.

Camilla: The second strategy is we felt like their groups... The grouping was too big, and Lew was even just like, I should've put them in smaller groups. (PCK, M)

Yolanda: How many were in each group? (PCK, M)

Camilla: Well there were really four groups. (PCK, M)

Janice: Four groups. (PCK, M)

Janice: And so sss. (PCK, M)

Camilla: So there was like eight. (PCK, M)

Yolanda: That's pretty big. (PCK, M)

Janice: Ya. (PCK, M)

Janice: Well it's some... seven, six or seven. (PCK, M)

Camilla: No, maybe six or seven in each group. (PCK, M)

Janice: And then the problem, in fact I had even jotted it down, that most of the boys were having too much fun just playing with the manipulatives... (PCK, M)

Camilla: Ya, and the groups... (PCK, M)

Janice: ...and not, you know, even working. (PCK, M)

The topic of concern in this conversation was that the groups were so big that the children were playing with the manipulatives instead of focusing on the task. Playing with manipulatives instead of working on the task was a distraction that needed to be diminished and therefore the above conversation was coded as management. The sentences are examples of PCK because the participants were discussing an appropriate group size for an effective experience with mathematics manipulatives.

Interruptions were common in the conversations between participants. Notice that Camilla interrupted Janice by saying "so there was like eight." Janice's sentence was cut short: "And so sss." It is assumed that since the rest of the conversation was about management that Janice's sentence would have also pertained to management.

Nearly 2% (1.6%) of the sentences made during the lesson study pertained to management (see Table 4). Nearly 44% (43.5%) of the management sentences are coded as PCK, M (see Table 6).

Comparison

Table 4 summarizes the quantitative findings of the coding described in this chapter without the application of relational and instrumental codings. The sum of the percentages is greater than 100. The reason for this occurrence is because multiple codes were applied to many cells. It was common, for instance, for a sentence coded as discussion to also have another code. This occurrence almost explicitly accounts for the overall discrepancy.

Tables 5 and 6 display the qualitative findings associated with the relational and instrumental coding as applied to SMK and PCK respectively.

Table 4

Coding Results Without Application of Relational and Instrumental Codes

Sentence Code	*Percent of Overall Occurrence*
Subject Matter Knowledge (SMK)	2.2
Pedagogical Content Knowledge (PCK)	73.6
Management (M)	1.6
PCK Management (PCK, M)	0.7
Pedagogy (P)	10.1
Discussion (D)	12.0
Personal	9.1
Protocol	2.8
No Code	.7

Table 5

Relational and Instrumental Coding Applied to SMK

SMK Sentence Code	*Percent of Overall Occurrence*	*Percent of Overall SMK*
Relational SMK	1.2	53.5
Instrumental SMK	0	0

Table 6

Relational and Instrumental Coding Applied to PCK

PCK Sentence Code	*Percent of Overall Occurrence*	*Percent of Overall PCK*
Relational PCK	7.8	10.6
Instrumental PCK	1.5	2.1

Chapter 5 – Discussion and Conclusion

The purpose of this chapter is to discuss inferences associated with the quantitative results listed in Chapter 4 and to state some overall conclusions about the research of this thesis. The inferences associated with the data are as follows: (1) focusing on the research lesson goal motivated PCK conversation, (2) instrumental PCK conversation was associated with anticipated student mistakes, (3) relational PCK conversation occurred as the participants created a lesson promoting student understanding. Additionally, some findings will be presented with respect to the participants' preparation to orchestrate a discussion.

Goals and Pedagogical Content Knowledge

While preparing the research lesson, focus on the research lesson goal motivated PCK conversation. Recall that for the purposes of this thesis, PCK is defined as a blending of content and pedagogy. It takes into account student thinking, how a topic should be organized and presented, and classes of student errors. It can be thought of as SMK applied to teaching where SMK is knowledge of concepts, algorithmic operations, connections between procedures, and student errors.

Example 1: Camilla's Question. The lesson study goal—improvement of math self-esteem through subtraction of mixed fractions—first used the problem $7\frac{1}{8}-1\frac{3}{4}$. As the participants were finalizing their goal by writing it on paper, Camilla asked "how are we engaging students to improve self-esteem with this lesson?" (PCK). Lew responded that it was "through allowing think time, individual think time, for allowing and demanding individual think time” (PCK). Janice inserted “Then group think time” (PCK). Camilla’s question was coded as PCK because she was inquiring about how their mathematics lesson should be taught. Lew and Janice’s responses to Camilla’s questions were coded as PCK because they were considering how to organize the mathematics lesson to allow individuals and groups time to think about a problem.

The lesson study goal—improvement of math self-esteem through subtraction of mixed fractions—first used the problem $7\frac{1}{8}-1\frac{3}{4}$. As the participants were finalizing their goal by writing it on paper, Camilla asked “how are we engaging students to improve self-esteem with this lesson?” (PCK). Lew responded that it was “through allowing think time, individual think time, for allowing and demanding individual think time” (PCK). Janice inserted “Then group think time” (PCK). Camilla’s question was coded as PCK because she was inquiring about how their mathematics lesson should be taught. Lew and Janice’s responses to Camilla’s questions were coded as PCK because they were considering how to organize the mathematics lesson to allow individuals and groups time to think about a problem.

Lew explained that the importance of individual think time before group time allowed students to “clarify thinking before you have kids talk out loud to the group” (PCK). He also noted that in “the whole you get the chance to share your thinking with somebody else” (PCK), and that group work allows “people to show their thinking in different ways” (PCK). Implicit in each of these sentences is the idea of shared thinking. To Lew, their research goal would be met by students formulating and communicating ideas. Student thinking and organization of a class so that thinking may be shared are components of PCK.

Lew and Janice also suggested that the way they encourage students would motivate them to think individually and work together in groups. Lew stated “We're not praising the final answer” (PCK), to which Janice responded “No, we're doing the participation” (PCK). Again, the organization of the lesson, a component of PCK, relied upon the students’ shared thinking.

Individual and shared thinking were essential to the research lesson goal but Janice and Lew recognized that they were not sufficient. Students were to learn how to subtract mixed fractions.

Janice: Ok, through positive praise of individual and group participation, students will… (PCK)

Lew: Explore possible ways to solve… (PCK)

Janice: To solve subtraction of mixed numbers? (PCK)

Janice and Lew anticipated that through the conversations, students could explore mathematical ideas central to the research lesson goal. Designing a lesson for student exploration is a concept related to PCK and is also an essential aspect to the *explore* component of the CMI framework.

Example 2: Graham Crackers and Goals. In Example 1, Camilla asked Lew and Janice how the lesson preparation was meeting the research lesson goal. In this example, Camilla offered her own suggestion about how to meet their goal. While searching their manuals for appropriate tasks and lesson topics, Camilla called Lew and Janice’s attention to a classroom activity using graham crackers. Here Camilla introduces the graham cracker again as a student motivator when she says, “um, the only reason why I even just wanted to look back at that is ('cause I think the question stinks what they're asking with that, but) it just sounds more fun" (PCK).

Camilla felt that the fun associated with a graham cracker as a manipulative would get the students more involved than if they simply placed a mathematics problem on the board:

> Because I was just thinking like I don't know if my math self-esteem is going to be improved if my teacher throws somethin' up on the board and says do it. Like, I I think I would feel intimidated and, you know what I'm sayin'? (PCK)

Lew noted that a graham cracker was partitioned into thirds or fourths. That caused them to consider other manipulatives that one could use to represent $7\frac{1}{8}-1\frac{3}{4}$. First they considered paper folded into fourths and then eighths. They considered using brown paper so that it would look like a graham cracker. Circles partitioned like pizza slices were also considered. As Camilla, Janice, and Lew considered possible manipulatives and stories to match the manipulatives, they were using PCK. That is to say Camilla and Janice were considering how the topic of subtraction of mixed fractions should be presented or organized to best motivate student interest and motivation for the *explore* component of the CMI framework.

As the creation of the research lesson continued to progress, the participants no longer explicitly spoke of the lesson study goal. However, as seen above, when the goal of the lesson study was the focus of the conversation, PCK conversation naturally followed. While a correlation between goal centered conversations and PCK is evident, a stronger inference could be made with more examples.

Instrumental PCK and Student Errors

Though the instances of instrumental PCK codes are relatively few (1.5% of the overall sentences), the pattern associated with their occurrences is noteworthy. In contrast to the relational PCK conversations occurring during episodes where the participants discussed methods to increase student understanding, instrumental PCK conversations occurred only when the participants discussed classes of student errors.

On January 24th when the participants were invited to think about anticipated student responses to their initial task, various instrumental mistakes which might be manifested during the *explore* phase of the CMI framework were proposed. Two of them are presented for consideration. Recall, that on the 24th of January the research lesson was still associated with the task $7\frac{1}{8} - 1\frac{3}{4}$.

When asked to anticipate student responses, Camilla said "what if they just do it this way and they go four times two is eight. Three times two is six," as she rewrote $1\frac{3}{4}$ as $1\frac{6}{8}$. Janice finished Camilla's thought when she jumped in and said "six and five-eighths." To get this answer, Janice and Camilla subtracted the whole number, one, from the whole number, seven, and subtracted the fraction, one-eighth, from the fraction, six-eighths, to yield six and five-eighths. As detailed in Chapter 4, this episode was coded as instrumental PCK because the participants focused on student errors which would result from not knowing the reasoning behind an algorithm.

In another anticipated student response to the same problem, Camilla said "we could even get eight and seven-eighths if they are just ignoring the sign. Do you know what I'm saying? Really" (PCK, I). To that Lew responded "They ignore the sign all the time" (PCK, I).

The PCK coding is applicable because Camille and Lew were taking into account student thinking. It may not be appropriate to associate instrumental errors with mistaken signs in all situations. In this situation, however, the sum of the mixed fractions would yield a larger number than that either of the initial fractions. By not recognizing the discrepancy in the proposed answer one would not be reasoning about the answers. Therefore it would be an appropriate assessment that such a student was using a rule without reasoning.

When Camilla, Janice, and Lew finished brainstorming about possible student solutions, they had eight different responses listed on the handout by Dr. S. Six of the eight errors were attributed to potential student instrumental mistakes.

Relational PCK and Student Understanding

In contrast to the correlation between instrumental PCK and anticipated student mistakes, there was a correlation between relational PCK and desired student understanding. A coding of relational PCK signifies a sentence where one of the participants focused on a student's understanding of what he/she was doing and why. When Camilla, Janice, and Lew focused on student learning of subtraction of mixed fractions their PCK conversations were relational in nature.

When the participants had first decided to focus the research lesson on subtraction of mixed fractions, Lew and Janice thought that an appropriate launch might be to display a problem with its answer on the board and elicit student justification of the answer:

Janice: Well so, ok, the thing is if you put this up on the board and you put that answer then they would have to show how they got the answer to show, and then, or…" (PCK, R)

Lew: And how they figured it out. (PCK, R)

Janice:And how they figured it out. (PCK, R)

Janice and Lew suggested that students should "show" how they got the answers to the problem as part of the *explore* phase of the CMI framework. By suggesting that students show and explain their reasoning they were inviting student understanding. This type of conversation fit the criteria for relational PCK.

Later, after selecting the problem $3\frac{1}{6}-1\frac{5}{6}$ for a launch, Camilla, Janice, and Lew discussed what their roles would be during the launch portion of the research lesson. They thought that interaction with the students during this time would afford them opportunities to build student understanding through questioning:

Camilla: And then then that's where we have all of our questions, our questions, our probing questions, like… (PCK, R)

Lew: Right. We'll walk around and… monitor, probe. (PCK, R)

Camilla: You know, ya. (PCK, R)

Camilla: You know just like those: What what were you thinking? Can you explain that to me? Prove it to me. Can you can you explain what she just said? (PCK, R)

Lew: Then we'll let the kids share. (PCK, R)

Lew and Camilla suggested that the students share their thinking with the teachers and with their classmates. The types of questions they were suggesting asking of the students were ones eliciting student reasoning and justification. For example, Camilla suggested having the students "prove" an answer. This type of teaching is relational in nature.

The use of manipulatives was also frequently suggested to invite student understanding. In the following example, Camille and Janice wanted students to explain the solution for $3\frac{1}{6}-1\frac{5}{6}$ using multiple representations. As Camilla explained, "multiple ways of solving are gonna, going to help to push them further" (PCK). As an example of how that pushing might occur she volunteered the following scenario:

Camilla: Find a new way. Do it with a picture. Do it with a square picture. Do it with a circle picture. Do it with a … (PCK, R)

Janice: Manipulatives. (PCK, R)

Camilla: ya. (PCK, R)

Camilla and Janice were interested in how the lesson could be taught so that students would understand the concepts through pictures and manipulatives. This is an example of relational PCK.

Camilla was also interested in how her students would explain their reasoning using the manipulatives as tools. In the following quote, Camilla explained how she would invite her students to justify their reasoning to each other during the *explore* component of the CMI framework:

> So after they do it individually, then I'm gonna, I'm gonna invite them to explain it to their group, explain it, ask questions, justify, prove your answer, prove your thinking. Whether you do it through manipulatives, pictures, whatever. (PCK)

One of the methods Camilla suggested that her students explain their reasoning was through manipulatives. This conversation is a demonstration of PCK because Camilla talked about how a lesson should be organized. It is a demonstration of relational PCK because Camilla shared how she would invite students to explain to each other how and why $3\frac{1}{6}-1\frac{5}{6}=1\frac{1}{3}$. Camilla was not very specific in the types of justifications or reasoning she anticipated from her students. This does not necessarily indicate that she had no relational PCK in this area, but was rather working at a different level of relational PCK.

It is worth noting that student's conversations and student's use of manipulatives are two of Hiebert et al.'s (1997) critical features of classrooms, features that "can be thought of as a set of guidelines that teachers can use to move their instruction toward the goal of understanding" (p. 7).

According to Hiebert et al. (1997), "tools play a kind of intermediary role for developing meaning" (p. 53-54). By suggesting that the students reason with manipulatives, Camilla, Janice, and Lew were setting the stage for student understanding. Camilla, Janice, and Lew even altered their launch and task so that he manipulatives would facilitate student understanding of mixed fraction subtraction.

Another of Hiebert et al.'s (1997) critical features of classrooms is the social culture of the classroom. In describing the social culture of the classroom they state the following:

> Observations of how mathematical communities solve problems show both individual and group efforts. Group efforts require a great deal of communication. Assumptions about what things mean must be agreed on,

> assertions or conjectures are made, methods of solutions are proposed and defended, challenges are usually offered, and discussions are held about the soundness and accuracy of solutions. These activities are all part of doing mathematics and all involve intense communication and social interaction. Classrooms that experience some form of these activities reveal to their participants what doing mathematics is all about. (p. 43-44)

According to Hiebert et al. (1997), the participants' efforts to invite students to explain their reasoning to each other would reveal to the students part of what "math is all about." It is not clear whether Camilla, Janice, and Lew were interested in revealing to their students "what math is all about" (p. 44). It is clear, however, that their desire to invite student understanding inspired methods of instruction which encouraged relational understanding.

Discussion Component of the CMI Framework

The six components of the framework are the (1) Launch, (2) Explore, (3) Discuss, (4) Extend, (5) Practice, and (6) Demonstration of Understanding. The classroom discussion is one of those components. Under the CMI framework, it is likely that a class discussion could account for up to one-third of a mathematics lesson (See Appendix A). This section details some observations about the participant's preparation for a classroom discussion.

During the lesson study process the teachers spoke at length about an initial launch and subsequent student exploration. Each of the examples presented in this chapter represent conversations about the launch or the explore component of the CMI framework. Many of the participants' conversations were focused on choosing an initial launch and subsequent student exploration that it was unclear how much preparation was made for the discussion component of the CMI framework.

Both the quantity and the quality of the "discussion" conversations are noteworthy. Twelve percent of all the participants' sentences pertained to the discussion component of the research lesson. Eleven percent of the discussion related sentences occurred before the first iteration of the research lesson and 89% occurred after the first iteration.

The large increase in the focus on discussion component after the first iteration of the research lesson indicates that the teachers needed to spend more time on discussion preparation. The increased focus on the orchestration of a class discussion after the first iteration of the research lesson may be attributed to two factors. First, it is possible that after watching Lew teach the first research lesson to his class, Janice and Camilla realized that they needed to devote more time to preparation of the discussion. Second, on the date designated for research lesson revision, Dr. S distributed a worksheet designed to motivate preparation for a classroom discussion (see Appendix F).

It is not known whether the participants would have engaged in as much conversation about the discussion component of the research lesson without the aid of the worksheet. They did, however, engage in a significant amount of conversation because of it. It may be observed that Dr. S's intervention had a profound impact on the nature of the research lesson participation. Similar instances of intervention occurred when the participants were asked to consider anticipated student responses earlier in the research lesson creation. Such intervention is common in traditional Japanese lesson study:

> The central force in a research lesson is the teacher or group of teachers who develop the lesson. Observing teachers also play an important role. A research lesson would not function as professional development without these two groups of players. A third player who sometimes participates is the invited outside specialist. For mathematics lesson study, this role may be filled by a teacher, principal, or university mathematics educator who is known for his or her expertise in mathematics teaching. The invited specialist may comment on the lesson, provide advice as the lesson is developed, teach a research lesson, or provide a summary at the faculty colloquium. (Lewis, 2002b, p. 32)

Dr. S acted as an outside specialist when he invited all of the participants to give additional thought to the discussion components of their research lessons.

Though Camilla, Janice, and Yolanda conversed about the orchestrating of the classroom discussion, their conversations lacked some needed details. They planned very broadly to conduct a discussion, but they did not plan some of the more minute and important details. The following conversation exemplifies their preparation for the transition from the explore component to discussion component of the CMI framework.

Camilla: So after they do it individually, then I'm gonna, I'm gonna invite them to explain it to their group, explain it, ask questions, justify, prove your answer, prove your thinking. (PCK, R)

Camilla: Whether you do it through manipulatives, pictures whatever. (PCK, R)

Camilla: Then do I say, "take what you guys have and put it on an overhead?" or do I say, “ooh you know what? I like that one. Put that on an overhead" (PCK, D)

Camilla: I want to talk about that one. (PCK, D)

Janice:I think you have 'em all put it on an overhead. (PCK, D)

Camilla: So they all feel responsibility. (PCK, D)

Janice: Feel responsible, ya. (PCK, D)

Camilla: So take the ideas that you have… (PCK, D)

Janice: But then, but then you'll decide… (PCK, D)

Camilla: Who I want to do it. (PCK, D)

Janice: Ya who you'll want to come up and share. (PCK, D)

In preparation for the classroom discussion Camilla and Janice planned to have all the children place their answers on overhead transparencies. Camilla would then be able to select various students to present their solutions to the class. Camilla acknowledged that there would be order to her choice of presentations when she stated: “I'll invite specific students that I am watching for how I want to guide my discussion.” Following this excerpt there were a few more sentences about the need for plenty of overhead transparencies so that only one answer would be written per transparency. It was clear that Camilla anticipated that there would be order in her choice of presenters. She reiterated this point when she said: “I'll be inviting everyone to be participating in whatever that student is presenting and then start the whole thing over again with the new worthwhile mathematical task.”

Though Camilla did not mention what that order would be, she may have intended to follow the guidelines she had formulated with Lew and Janice two weeks previously when they stated that pictorial presentations should take precedence over procedural answers:

Lew: Why don't we just start with pictures? (PCK, D)

Camilla: Same. (PCK, D)

Janice: Pictures, pictures, exactly (PCK, D)

Camilla: I definitely think pictures because once they make a connection with pictures then it's easier to do it the other way… (PCK, R, D)

Camilla: Which so they learn that the other way is more efficient, so they're able to do that better. (PCK, R, D)

It was clear that the participants planned for a smooth transition from an exploration to a discussion. They also had a governing guideline to present student pictorial responses before student procedural responses. After the first iteration of the lesson, Janice and Camilla also recognized that students needed less intervention from them when presenting their solutions.

Both Janice and Camilla felt that Lew was too quick to aid students with their presentations. Janice and Camilla, in reaction to Lew's help with an incorrect solution, said:

Camilla: So now for me, maybe I would've just kept my mouth shut and kept going and said… (PCK, D)

Janice: Questions questions. (PCK, D)

As a result of their experiences in Lew's classroom, Janice and Camilla decided that they would invite more correct solutions to be presented and more questions from the observing students. Their hope was that the students would help answer each others' questions and clarify each others' misconceptions. It was clear to them that there were student misconceptions in Lew's classroom. Many of Janice, Camilla, and Yolanda's conversations during the lesson revision were about different student misconceptions.

It was in clarification of student misconceptions that the participants' discussion preparations were lacking. Though they foresaw certain classes of student errors and certain correct representations (as noted in the previous sections), they did not discuss explicitly how they would use the classroom discussions to aid the students with misconceptions. Similarly, they did not state specifically which correct solutions they wanted presented, nor did they state which incorrect solutions they wanted presented.

This is not to say that their orchestrated classroom discussions were not useful. Based on the observation of the videotaped research lesson, there was evidence that students were actively engaged in dialogue, listening to each other, and striving to understand each others' solutions.

Conclusions

Much is still unknown about the effects of lesson study on participating U.S. teachers or their students. It is understood that U.S. teachers need appropriate and adequate professional development opportunities in order to grow as professionals (NCTM, 2000; Loucks-Horsley et al., 2003) and that teachers "often do not receive the support they need to keep their pedagogical skills and content knowledge current"

(NRCC, 2001). The results from this study contribute to the research literature on year-long professional developments by demonstrating how teachers' mathematical discussions are fostered as they participate in lesson study and ground their efforts in the CMI framework.

More and specific studies would need to be conducted in order to determine if lesson study addresses the pedagogical and content knowledge needs of teachers. However, this study does lend some insight into what teachers discuss when engaged in a CMI framework lesson study, and how those discussions may be used to meet the professional development needs of teachers.

Lesson study with teacher efforts grounded in the CMI framework is a vehicle to motivate PCK oriented conversations. This thesis shows that more than 70% of the conversation among one group of teachers collaborating on a research lesson addressed student thinking and student learning. Some of their PCK conversation focused on anticipated student solutions, diverse methods of instruction, and student motivation. This type of sustained experience could help to keep pedagogical skills current.

It may be argued that teacher collaboration, in general, may motivate PCK conversations. The results of this thesis seem to indicate, however, that the goal aspect of lesson study may be integral in focusing teachers on student thinking. It helped motivate the participants to give a more dynamic and adaptive lesson because they considered means to make their lesson accessible and the content understandable to all students.

One of the implications of this study is that the practice of lesson study in conjunction with the CMI framework focuses the participating teachers' efforts on student learning. The participants of this research had a goal that was centered on student mathematical needs and conversation naturally supported PCK centered conversations. Future research could be done to verify the effects of goals on teacher conversations by analyzing teacher conversations motivated by instrumental goals.

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Appendix A

CMI Framework Template

Comprehensive Mathematics Instruction (CMI) Framework

Teacher Name: Grade: Topic:

LEARNING GOAL:

STATE STANDARD:

CONCEPT DEVELOPMENT			ADDITIONAL INSTRUCTION	
LAUNCH	**EXPLORE** Anticipated Student Thinking	**DISCUSS** Anticipated Orchestration	**EXTEND**	**PRACTICE**
5-10 Minutes	20-30 Minutes	15-20 Minutes	Outside of Instruction	10-15 Minutes
MATERIALS	DEMONSTRATION OF UNDERSTANDING			

Appendix B

MIC Description of the CMI Framework

The CMI framework is a lesson-planning tool that gives structure to classroom instruction while incorporating essential characteristics of mathematics pedagogy. The framework comprises six components: Launch, Explore, Discuss, Extend Understanding, Fluency, and Demonstration of Understanding. These six components are grouped according to different aspects of the overall instruction as shown below:

- Concept development
- Launch
- Explore
- Discuss
- Additional Instruction
- Extend understanding
- Fluency
- Demonstration of Understanding

The framework is not written as a day by day "lesson plan format" but rather as a structure to develop a concept over time. Although there are time intervals listed on the framework, they do not imply a set 50 minute time frame to cycle through the framework each day. Rather, they suggest the relative importance that should be placed on each portion of the concept development phase of the framework.

Concept Development

This portion of the framework is different from the pedagogy that is typically found in mathematics classrooms and can be difficult to implement for many teachers. It is, however, the most important part of the framework because it focuses on allowing students to develop their own understanding of mathematical concepts and procedures instead of just memorizing algorithms. The phases of concept development are launching a mathematical problem, allowing students to explore the solutions to the problem and then discussing the various student strategies as a way of making connections between different representations and concepts.

Launch. During the Launch the teacher introduces a task and asks a question or series of questions that invites the students to explore the task. This task has a clear conceptual purpose tied to a standard core objective but is designed to allow multiple solutions or multiple paths to one correct solution. The launch may also revisit a previously posed problem in order to clarify, refine, extend, continue, or summarize a previous days exploration. Another source of a launch could be a student-generated problem or a task targeting a student need that the teacher identified during the informal assessment portion of the demonstration of understanding. Tasks are also designed to help students make connections with previously learned mathematics.

The role of the teacher during the launch is to invite students to learn by posing questions that can lead to a specific mathematical objective. The student's role is to actively listen and ask any clarifying questions before they engage in the exploration of the posed task.

Explore. During the Explore phase the students work individually or in small groups to respond to the questions. The teacher carefully observes the students' exploration, asking questions to probe and clarify their thinking. In preparation for the explore phase of the lesson, the teacher anticipates the types of thinking that students might use when engaging in the posed launch.

Students will use different problem solving strategies as well as a variety of representations (charts, tables, diagrams, pictures, manipulatives, technology, etc.) to solve the problem. They may also work individually or in groups to explore this problem. In the process of explorations, students will translate among different mathematical representations, make connections and build their own understanding of mathematical concepts.

The teacher will facilitate the exploration by asking clarifying questions to probe and push the students to think deeply. The teacher will also take note of the various methods used by the students and mentally compare them to those that were anticipated during the preparation of the lesson. This informal assessment is used in anticipation of the discuss phase of the lesson.

Discuss. The teacher orchestrates the discussion by purposely selecting different students to present their results to the class based on the informal assessment during the explore phase of the lesson. The selected student work may be ordered by level of complexity to develop connections between different strategies. The teacher may even select incorrect methods in order to illustrate a common misconception. A broad range of solutions might also be selected so that most students can identify with at least one solution, and so they can learn from other solutions.

Students not only share their solutions but are also expected to explain the reasoning behind their results and the mathematical representations that they have used. The other students in class actively listen, by asking questions, to understand, clarify and verify their peers' mathematical reasoning. This conversation about solution strategies and reasoning requires students to use a common mathematical vocabulary and evaluate strategies based on correctness, efficiency, elegance, originality, and clarity.

The teacher will facilitate the discussion and initially the may have to model the language of a mathematician and the types of questions that will help clarify the mathematical reasoning behind the strategies. The teacher may also help students understand and decide upon the criteria for judging strategies and conclusions. Once these criteria are agreed upon, best strategies may be chosen based on peer consensus.

In the end, the exploration and subsequent discussion of the launch will lead the students to conclusions, generalizations, and understandings of mathematical principles that can be applied to everyday mathematical problems.

Additional Instruction

Extend Understanding. As students explore a launch to build understanding of a mathematical concept, they may ask "what if" or "I wonder" questions that would naturally extend the existing problem or concept. They may also demonstrate a fragile understanding of the concept at hand. It is through these types of informal assessment that the teacher identifies topics/questions/ concerns appropriate for a future lesson launch They may initiate another launch-explore-discuss cycle to address the extension or to deepen a fragile understanding.

The teacher can extend or deepen understanding by posing a related problem in a different context than the original launch. This may be done as a homework problem or as another launch-explore-discuss cycle. After students have done multiple problems related to the same concept, they will begin to make connections and see patterns in the solutions and strategies that have been employed. These patterns allow students to develop their own strategies and algorithms based on understanding. This understanding of concepts and algorithms builds toward the students developing fluency in reasoning and computation.

Fluency. The Fluency phase is individual work that focuses on student fluency of reasoning and basic skills. During this phase students have the opportunity to develop automaticity by practicing their computational fluency of basic facts. It is critically important that this fluency is not based on memorization but rather on conceptual understanding.

In addition to fluency of computational procedures, the students can become fluent in their use of reasoning to solve problems. They can become fluent in the strategies that they use in the explore phase of the concept development. They can become fluent in their use of different representations and the connections between them. It these later fluencies that are developed by solving worthwhile problems and not just memorizing procedures.

Demonstration of Understanding. The demonstration of understanding is a strand of the framework that is ongoing throughout the other components. The teacher is constantly assessing student thinking and understanding throughout the concept development and additional instruction phases using a variety of measures and methods. Often student understanding is only assessed formally using summative evaluation like homework, quizzes or tests. However, students are demonstrating their understanding throughout the launch-explore-discuss cycle. They do so through verbal and/or written communication using multiple representations (i.e. pictures, numbers, words, tables, graphs, manipulatives, technology, proofs, etc.). Thus, as students solve problems individually and in groups, it becomes important for teachers to monitor conversations, to observe individual student writing and use of representations and to ask clarifying questions of individuals and groups.

Requiring students to explain their reasoning in all phases of instruction is provides additional opportunities for them to demonstrate their understanding. As students conclude their work on a launch, asking them to make generalizations or draw conclusions is also an effective way to assess their thinking.

More formal, summative types of assessment are journals, constructed responses, performance tasks or portfolios.

Appendix C

Anticipated Coding	
Code	Description
Subject Matter Knowledge (SMK)	Concepts Algorithmic operations Connections between procedures Student errors
Pedagogical Content Knowledge (PCK)	A blending of content and pedagogy Takes into account student thinking How a topic should be organized How a topic should be presented Student Errors SMK applied to teaching
Relational Understanding (R)	What one is doing and why
Instrumental Understanding (I)	Rules without reason
Pedagogical Issues (P)	Teaching not specific to mathematics
Classroom Management (M)	Classroom Control Maintain Safety Diminish Distractions

Appendix D

Additional Coding	
Code	Description
Personal	"Concerning a particular person and his or her private business, interests, or activities" (Pickett et al., 2000)
Protocol	Understanding directions What is expected of participants
Discussion (D)	Sentences about the discussion portion of the research lesson.

Appendix E

Subtraction of Fractions Task 1

January 24, 2005

Given the problem $2\frac{1}{4}-1\frac{3}{4}$ and any tools that you have available on your table, come up with as many anticipated student solutions to this problem as you can. Share these responses with the other members of your group.

Appendix F

Orchestrating the Discussion
Issues to Consider

1. If many students are struggling to make adequate headway during the exploration part of the lesson do you stop and tell or let them struggle or start the discussion or ???

2. When do you stop the exploration part of the lesson and begin the discussion?

3. When a student shares a misconception or incorrect method as part of the discussion, does the teacher step in and clarify immediately or let other students continue to share and monitor whether or not the misconception has been adequately addressed?

4. When the students have shared their solutions and there is no clear consensus as to which one is correct, how do you nudge them to the correct one so that you can move ahead?

5. Once you know that general types of solutions that are likely to emerge, what order are you going to have those solutions shared and why?

6. What are the key questions that you are going to ask during the explore portion of the lesson and how will they be worded and how will they be worded?

7. How much do you focus on or help the students organize the various solution methods?